

نام و نام خانوادگی: شماره: کلاس: پاسخنامه تشریحی

الف) $a_n = \frac{1}{2}, 1, 2, \dots \Rightarrow q = 2$ $a_{10} = a_1 \times q^9 \Rightarrow \frac{1}{2} \times 2^9 = 2^8 = 128$

ب) $\frac{a_{12}}{a_{12}} = \frac{a_1 \times q^{14}}{a_1 \times q^{12}} = q^2 = 2^2 = 4$

ج) $\begin{cases} a_2 = a_1 q^1 \\ a_{10} = a_1 q^9 \end{cases} \Rightarrow \sqrt{a_2 \times a_{10}} = a_6 = a_1 \times q^5 \Rightarrow \frac{1}{2} \times 2^5 = 2^4 = 16$

د) $128 = a_1 \times q^{n-1} \Rightarrow 128 = \frac{1}{2} \times q^{n-1} = \frac{1}{2} \times 2^{n-1} \Rightarrow 2^{n-1} = 256 \Rightarrow n-1 = 8 \Rightarrow n = 9$

$\begin{cases} a_8 = 94 \\ a_{10} = 12 \end{cases} \Rightarrow \frac{a_8}{a_{10}} = q^2 = 8 \Rightarrow q = 2$

$a_{10} = a_1 \times q^9 \leq a_8 \times q^2 \Rightarrow 94 \times 2 = 188$

الف) $a_1 \times a_2 \times a_3 \times a_4 \times a_5 = 243 \Rightarrow (a_3)^5 = 243 \Rightarrow a_3 = 3$

$a_4 = a_3 \times q$

ب) $a_1 \times a_5 = a_3 \times a_3 = 3 \times 3 = 9$

2^a و $2^b \Rightarrow (2^{\sqrt{2}})^2 = 2^2 \Rightarrow 2^a \times 2^b \Rightarrow 2^2 = 2^{a+b} \Rightarrow a+b=2$

طایفه حسابی $= \frac{a+b}{2} = \frac{2}{2} = 1$

$a_{11} \times a_{19} = a_7 \times a_{23} \Rightarrow (n+1)(1-n) = n^2$

$a_7 = 1-n$

$a_{19} = n$

$a_{11} = n+1$

$1-n^2 = n^2 \Rightarrow 2n^2 = 1$

$\frac{\sqrt{2}}{2} = \text{جواب}$ $\Rightarrow n^2 = \frac{1}{2} \Rightarrow n = \pm \frac{\sqrt{2}}{2}$

$$\begin{aligned} a_1 + a_2 &= a(q + 1) = 12 \\ a_1 + a_4 &= a(1 + q^3) = 18 \end{aligned} \Rightarrow \frac{a(1+q^3)}{a(q+1)} = \frac{18}{12} = \frac{3}{2} = \frac{(1+q)(1-q+q^2)}{2(q+1)}$$

$$\Rightarrow \frac{1}{2} = (q^2 - q + 1) \Rightarrow 2q = 2q^2 - 2q + 2 \Rightarrow 2q^2 - 4q + 2 = 0 \Rightarrow q^2 - 2q + 1 = 0 \Rightarrow (q-1)^2 = 0 \Rightarrow q = 1$$

اذا $q = 1 \Rightarrow a = 1 \Rightarrow a_n = 1, 1, 1, \dots$ ✓
 مع $q = \frac{1}{2} \Rightarrow a = 2 \Rightarrow a_n = 2, 1, \frac{1}{2}, \dots$ ✓

$$a_1 \times a_2 \times a_3 = 27 \Rightarrow a_1^3 = 27 \Rightarrow a_1 = 3$$

$$a_1 + a_2 + a_3 = 12 \Rightarrow a_1 + a_2 = 10$$

$$\frac{a_1}{q} + a_2 + a_3 = 12 \Rightarrow \frac{3}{q} + 3 + 9 = 12 \Rightarrow \frac{3}{q} = 6 \Rightarrow q = \frac{1}{2}$$

مع $q = \frac{1}{2} \Rightarrow a_1 = 9 \Rightarrow a_n = 9, \frac{9}{2}, \frac{9}{4}, \dots$ ✓

مع $q = 2 \Rightarrow a_1 = 1 \Rightarrow a_n = 1, 2, 4, \dots$ ✓

$$a_n = 2, 4, 8, \dots$$

ا) $S_{10} = a_1 \left(\frac{q^{10} - 1}{q - 1} \right) \Rightarrow 1 \left(\frac{2^{10} - 1}{2 - 1} \right) = 2^{10} - 1 = 1023$ ✓

ب) $P_{10} = (a_1 \times a_{10})^{10} = (a_1 \times a_{10})^5 = (1 \times 1023)^5 = 1023^5$ ✓

$a_n = \frac{a}{q}, a, aq, aq^2, aq^3, \dots$

$a_1^2 \times a_3 \times a_5 \Rightarrow a^2(q-1)^2 \Rightarrow \frac{a}{q}(q-1) = a(q-1)$ ✓

$\frac{a}{q}(q-1), a(q-1), aq(q-1), aq^2(q-1), \dots$

$a_n = a, a+d, a+2d, a+3d, \dots, a+(n-1)d$

$S_n = a + (a+d) + (a+2d) + \dots + (a+(n-1)d) = na + (d + 2d + \dots + (n-1)d)$
 $= na + d(1 + 2 + \dots + (n-1)) = na + d \frac{n(n-1)}{2}$
 $= \frac{n}{2} (2a + (n-1)d)$

$a_n = a, aq, aq^2, \dots, aq^{n-1}$

$S_n = a + aq + aq^2 + \dots + aq^{n-1}$ ✓

طريقة $\Rightarrow S_n q = aq + aq^2 + aq^3 + \dots + aq^n$
 $S_n(1-q) = a(1-q^n) \Rightarrow S_n = \frac{a(1-q^n)}{1-q} = a \frac{1-q^n}{1-q}$