

الف) $a_n = a_1 q^{n-1} \Rightarrow a_n = \frac{1}{2} \times (2)^{n-1} \xrightarrow{n=10} \frac{1}{2} \times (2)^9 = 2^8 = 256$

ب) $\frac{a_{17}}{a_{13}} = \frac{a_1 \cdot q^{17}}{a_1 \cdot q^{13}} = q^4 \xrightarrow{q=2} 2^4 = 16$

ج) واسطه هندسی (ج) = $\sqrt{a_4 \cdot a_{10}} = \sqrt{a_1 \cdot q^3 \cdot a_1 \cdot q^9} = \sqrt{a_1^2 q^{12}} = |a_1 \cdot q^6| \xrightarrow{\frac{a_1 > 0}{q > 0}} a_1 q^6 = a_7 = \frac{1}{2} \times (2)^{7-1} = \frac{1}{2} \times 2^6 = 32$

د) $128 = \frac{1}{2} \times (2)^{n-1} \Rightarrow 2^{n-1} = 256 \Rightarrow n = 9$ جمله ۹ ام

$\frac{a_8}{a_5} = \frac{a_1 \cdot q^7}{a_1 \cdot q^4} = q^3 = \frac{96}{12} = 8 \Rightarrow q = 2 \Rightarrow a_{10} = a_8 \cdot q^2 = 96 \times (2)^2 = 384$

$a_1 \times a_2 \times a_3 \times a_4 \times a_5 = 243 \Rightarrow (a_3)^5 = 243 \Rightarrow a_3 = 3$

$a_1 \times a_9 = a_3 \times a_7 \Rightarrow a_1 \times a_9 = 3 \times 3 = 9$

$(\sqrt{32})^2 = 2^{a+b} \Rightarrow 32 = 2^{a+b} \Rightarrow a+b = 5$ واسطه حسابی میانین $\frac{a+b}{2} = \frac{5}{2} = 2,5$

$a_1, a_2, a_3 \Rightarrow 1-x, x, x+1 \Rightarrow q^k = \frac{x}{1-x} \xrightarrow{\text{مجموع جملات فرد}} \frac{1}{\sqrt{2}} \checkmark$

$$\begin{aligned} a_1 + a_4 &= 28 \Rightarrow a_1 + a_1 \cdot q^3 = 28 \\ a_2 + a_3 &= 12 \Rightarrow a_1 \cdot q + a_1 \cdot q^2 = 12 \end{aligned} \Rightarrow \frac{(1+q)(1-q+q^2)}{q(1+q^3)} = \frac{28}{12} \Rightarrow \frac{1+q}{1+q^3} = \frac{7}{3}$$

جواب: بزرگترین عدد ۲۷

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$$3q^2 - 3q + 3 = 7q \Rightarrow (q-1)(q-9) = 0$$

$$3q^2 - 10q + 3 = 0 \Rightarrow q^2 - 10q + 9 = 0$$

روشن روی

$q = \frac{1}{3} \Rightarrow a_1 = 27$ و $a_2 = 9$ و $a_3 = 3$ و $a_4 = 1$

$q = 3 \Rightarrow a_1 = 1$ و $a_2 = 3$ و $a_3 = 9$ و $a_4 = 27$ بزرگترین عدد

$$a_1 + a_2 + a_3 = 13 \Rightarrow \frac{3}{q} + 3 + 3q = 13 \Rightarrow 3q + \frac{3}{q} - 10 = 0 \Rightarrow 3q^2 - 10q + 3 = 0$$

$$a_1 \times a_2 \times a_3 = 27 \Rightarrow \frac{a_2}{q} \times a_2 \times a_2 \cdot q = 27 \Rightarrow a_2^3 = 27 \Rightarrow a_2 = 3$$

$q = \frac{1}{3} \Rightarrow a_1 = 9$ و $a_2 = 3$ و $a_3 = 1$

$q = 3 \Rightarrow a_1 = 1$ و $a_2 = 3$ و $a_3 = 9$

$q = \frac{1}{3}$ یا $q = 3$: ۲ حالت

الف) $a_n = a_1 \cdot q^{n-1} = 2 \times 3^{n-1}$

$$S_n = a \frac{(q^n - 1)}{q - 1} = 2 \left(\frac{3^{10} - 1}{3 - 1} \right) = 3^{10} - 1$$

ب) $a_1 \times a_2 \times a_3 \times \dots \times a_{10} = a_1 \times a_1 \cdot q \times a_1 \cdot q^2 \times \dots \times a_1 \cdot q^9 = (a_1)^{10} \times q^{9+8+7+\dots+1} = 1$

$(a_1)^{10} \times q^{\frac{9 \times 10}{2}} = 1$ ضرب در اول $\Rightarrow 2^{10} \times 3^{45} = 1$

$$a_1, a_1 \cdot q, a_1 \cdot q^2, \dots \Rightarrow$$

برابر ۱ = قدر نسبت جیب

$$\frac{a_1 - a_1 \cdot q}{a_1(1-q)} + \frac{a_1 \cdot q - a_1 \cdot q^2}{a_1 q(1-q)} + \frac{a_1 \cdot q^2 - a_1 \cdot q^3}{a_1 \cdot q^2(1-q)} \Rightarrow$$

الف) $a_1, a_1 + d, a_1 + 2d, \dots, a_1 + (n-1)d = (a_1 + (n-1)d) + (a_1 + (n-2)d) + \dots \Rightarrow$

$$2S_n = (a_1 + a_1 + (n-1)d) + (a_1 + d + a_1 + (n-2)d) + \dots \Rightarrow 2S_n = n(2a_1 + (n-1)d)$$

$$\Rightarrow S_n = \frac{n}{2} (2a_1 + (n-1)d)$$

ب) $S_n = a + aq + aq^2 + \dots + aq^{n-1}$

$$qS_n = aq + aq^2 + \dots + aq^n$$

$$\Rightarrow qS_n - S_n = (aq - a) + (aq^2 - aq) + \dots + (aq^n - aq^{n-1})$$

$$\Rightarrow S_n(q-1) = \frac{aq^n - a}{q-1} \Rightarrow S_n = a \left(\frac{q^n - 1}{q-1} \right)$$

$$aq_1 \quad a_1 + d \quad a_1 + d \quad \dots \quad a_1 + (n-1)d = (a_1 + (n-1)d) + (a_1 + (n-2)d) + \dots \Rightarrow$$

$$\sum_{i=1}^n a_i = (a_1 + a_1 + (n-1)d) + (a_1 + d + a_1 + (n-2)d) + \dots \Rightarrow \sum_{i=1}^n a_i = n(a_1 + (n-1)d) \Rightarrow 1.$$

$$\sum_{i=1}^n a_i = \frac{n}{2} (2a_1 + (n-1)d)$$

$$\sum_{i=1}^n a_i = a + aq + aq^2 + \dots + aq^{n-1} \quad \Rightarrow \quad q \sum_{i=1}^n a_i - \sum_{i=1}^n a_i = (aq - a) + (aq^2 - aq) + \dots + (aq^n - aq^{n-1}) \Rightarrow$$

$$q \sum_{i=1}^n a_i = aq + aq^2 + \dots + aq^n$$

$$\sum_{i=1}^n a_i (q-1) = \frac{aq^n - a}{q-1} \Rightarrow \sum_{i=1}^n a_i = a \left(\frac{q^n - 1}{q-1} \right)$$