

$$a_n = \frac{1}{r}, 1, 2, \dots \rightarrow r = 2 \quad a r^{n-1} \rightarrow a_1 = \frac{1}{r} \times 2^9 = \boxed{2^{18}} = 286 \quad \text{الف}$$

$$\alpha = \frac{1}{r} \quad \text{ب}$$

$$\frac{a_1 r^9}{a_1 r^3} = r^6 = 2^6 = \boxed{64} \quad \text{ج}$$

$$\frac{a_1 r^3}{r}, \dots, \frac{a_1 r^9}{r} \rightarrow 4, 12, 36, 108, 324, 972, 2916, \dots$$

$$a_1 r^3 = 12 \quad a_1 r^9 = 286 \quad \frac{a_1 r^9}{a_1 r^3} = \frac{286}{12} = 1 = r^6 \Rightarrow r = 2 \quad \frac{r^3}{r} = 2 \text{ و } 12, 36, 108, \dots$$

$$a_1 r^3 = 12$$

$$a_1 r^9 = \frac{r}{2} \times 2^9 \Rightarrow 2 \times 2^9 = \boxed{1024}$$

$$a_1 \times 2^9 = 286 = 2^8 \times 2 \Rightarrow a_1 \times 2^9 = 2 \Rightarrow a_1 = \frac{2}{2^9} = \frac{1}{256} \quad \text{الف}$$

$$a_1 \times a_1 r^9 = a_1^2 r^9 \xrightarrow{a_1 \times a_1 r^9 = a_1 r^9} \boxed{2^9} = 512 \quad \text{ب}$$

$$\left(\frac{r}{r}\right)^r = \frac{r}{r} \times \frac{r}{r} \Rightarrow a+b=8 \rightarrow a=1, b=7 \rightarrow r = \sqrt[7]{2}, 1, 2, 4, 8$$

$$t_r = 1-n, t_v = n, t_{n+1} = n+1, t_v = t_r \times a^r, t_{n+1} = t_v \times a^r \Rightarrow n = (1-n)a^r$$

$$n+1 = n a^r$$

$$a^r = \frac{n+1}{n}, a^r = \frac{n}{1-n} \rightarrow \frac{n+1}{n} = \frac{n}{1-n} \quad (n+1)(1-n) = n^2 \rightarrow n+1 \leq n^2 - n = n^2$$

$$1 - n^2 = n^2 \Rightarrow 2n^2 = 1 \Rightarrow n^2 = \frac{1}{2} \rightarrow n = \pm \frac{1}{\sqrt{2}} \rightarrow a^r = \frac{n+1}{n} = \frac{1+\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = \sqrt{2}+1$$

$$a^r = \frac{n+1}{n} = \frac{1-\frac{1}{\sqrt{2}}}{-\frac{1}{\sqrt{2}}} = -(\sqrt{2}-1) \quad n = \pm \frac{1}{\sqrt{2}} \quad a^r = \frac{n+1}{n}$$

$$ar^3 = r^4 \quad r + ar^3 = 1^3$$

$$a = \frac{r^4}{r(1+r)} \quad \frac{1^3}{r(1+r)} (1+r^3) = r^4$$

$$1^3(1+r^3) = r^4 r(1+r) \Rightarrow r(1+r^3) = r^4 r(1+r)$$

$$r^4 - r^4 - r^4 + r^4 = 0 \quad (r-r^3)(r^2+r^2-1) = 0$$

$$\text{if } r = r^3 \rightarrow a = \frac{1^3}{r^4} = 1 \rightarrow \boxed{1, r, r, r^3}$$

$$\text{if } r = \frac{1}{r} \rightarrow a = \frac{1^3}{r^4} = r^4 \rightarrow \boxed{1, r, r, r^3}$$

$$\text{if } r = -1 \rightarrow \times \} \Rightarrow \boxed{r^4}$$

$$a_1 + a_1 r + a_1 r^3 = 1^3 \rightarrow a_1 + a_1 r^3 = 1$$

$$a_1 r^3 = r^4 \rightarrow a_1 r = r^4 = a_1$$

$$\frac{r^4}{r} + r + r^3 = 1^3 \Rightarrow \frac{1}{r} + 1 + r = \frac{1^3}{r} \Rightarrow$$

$$r + \frac{1}{r} = \frac{1}{r} \quad r^2 - 1 \cdot r + r = 0 \quad r = r \quad \underline{r} \quad \underline{r} = \frac{1}{r}$$

$$\text{if } r = r^3 \rightarrow a = \frac{r^4}{r^4} = 1 \rightarrow 1, r, r, r$$

$$\boxed{1, r, r, r}$$

$$\text{if } r = \frac{1}{r} \rightarrow a = \frac{r^4}{r^4} = 1 \rightarrow 1, r, r, r$$

$$\left. \begin{array}{l} a_1 = r \\ a_1 r = r^4 \\ a_1 r^3 = 1^3 \end{array} \right\} r = r^3 \quad r = r^3 \Rightarrow \boxed{r^3 - 1}$$

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$$a_1 \cdot r^3 = r^4 \rightarrow \boxed{1, r, r, r^3}$$

$$a, a r, a r^2, a r^3 \dots \rightarrow b_1 = a r - a = a(r-1)$$

$$b_2 = a r^2 - a r = a r(r-1)$$

$$b_3 = a r^3 - a r^2 = a r^2(r-1)$$

$$b_n = a r^{n-1} (r-1) \quad (n \geq 1)$$

$$\frac{b_{n+1}}{b_n} = \frac{a r^n (r-1)}{a r^{n-1} (r-1)} = r$$

$$a, a+d, a+2d, \dots, a+(n-1)d$$

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$$s_n = a + (a+d) + (a+2d) + \dots + [a+(n-1)d]$$

$$s_n = n[a + (n-1)d] \rightarrow s_n = \frac{n}{2} [2a + (n-1)d]$$

$$a, a r, a r^2, \dots, a r^{n-1}$$

$$s_n = a + a r + a r^2 + \dots + a r^{n-1} \quad a s_n = a r + a r^2 + \dots + a r^n$$

$$s_n - a s_n = a - a r^n \rightarrow s_n(1-r) = a(1-r^n) \quad a \neq 1 \rightarrow s_n = \frac{a(1-r^n)}{1-r} \quad \underline{s_n = \frac{a(1-r^n)}{1-r}}$$