نام و نام خانوادگی ...[name]... پاسخنامه تشریحی تکلیف شماره ...۱۴... کلاس ...دهم...

۱۸,۷۵

الف) $a_n = \frac{1}{2}, 1, 2, \dots \rightarrow q = 2 \qquad aq^{n-1} \rightarrow a_{10} = \frac{1}{2} \times 2^9 = \boxed{2^8} = 256$

$a = \frac{1}{2}$

ب) $\frac{\cancel{a}q^{16}}{\cancel{a}q^{12}} = q^4 = 2^4 = \boxed{16}$

۱,۵

ج)

د) $\underset{4}{a_1 q^4}, \dots, \underset{256}{a q^9} \rightarrow 4, 8, 16, 32, 64, 128, 256, \dots$

$n=9$

$128 = \frac{1}{2} 2^{n-1} \rightarrow n-1=8$

$a_1 q^4 = 12$

$a_1 q^7 = 96$

$\frac{a_1 q^7}{a_1 q^4} = \frac{96}{12} = 8 = q^3 \Rightarrow q = 2 \qquad \underset{a_1}{\frac{3}{4}}, \frac{3}{2}, 3, 6, 12, \dots, 96$

$a_1 q^9 = \frac{3}{4} \times 2^9 \Rightarrow 3 \times 128 = \boxed{384}$

۲

الف) $a_1 \times q^{10} = 243 = 3^5 \xrightarrow{q} a_1 q^2 = 3 = a_3$

ب) $a_2 \times a_1 q^4 = a_1^2 q^4 \xrightarrow{a_2 \times a_4 = a_1 \times a_5} \boxed{3^2} = \boxed{9}$

۲

$(\cancel{3}\sqrt{2})^{12} = \frac{2^9 \times 2^{\cancel{4}}}{2^{a+b}} \Rightarrow a+b = 5 \rightarrow a = 1, b = 4 \rightarrow q = 2\sqrt{2}, \quad 1, 2, 3, 4$

$\cdot 32$

۱,۵

$\text{واسطه حسابی} = \frac{a+b}{2} = \frac{5}{2} = \boxed{2.5}$

$t_3 = 1-n \quad t_7 = n \quad t_{11} = n+1 \quad t_7 = t_3 \times q^4 \quad t_{11} = t_7 \times q^4 \Rightarrow n = (1-n)q^4$

$n+1 = nq^4$

۱,۷۵

$q^4 = \frac{n+1}{n}, \; q^4 = \frac{n}{1-n} \rightarrow \frac{n+1}{n} = \frac{n}{1-n} \quad (n+1)(1-n) = n^2 \rightarrow n+1 = n^2 - n = n^{q_0}$

$1 - n^2 = n^2 \Rightarrow 2n^2 = 1 \Rightarrow n^2 = \frac{1}{2} \rightarrow n = \pm \frac{1}{\sqrt{2}} \rightarrow q^4 = \frac{n+1}{n} = \frac{1+\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = \sqrt{2}+1$

$q^4 = \frac{n+1}{n} = \frac{1-\frac{1}{\sqrt{2}}}{-\frac{1}{\sqrt{2}}} = -(\sqrt{2}-1) \qquad n = \frac{1}{-\sqrt{2}} \qquad q^4 = \frac{n+1}{n}$

فقط حالت + قابل قبول است زیرا به ازای $-\frac{\sqrt{2}}{2} = n$ جملات فرد با هم غیر هم علامت می‌شوند.

$a + ar^3 = 28$, $ar + ar^2 = 12$

$a = \frac{12}{r(1+r)}$, $\frac{12}{r(1+r)}(1+r^3) = 28$, $12(1+r^3) = 28r(1+r) \Rightarrow 3(1+r^3) = 7r(1+r)$

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$3r^3 - 7r^2 - 7r + 3 = 0$, $(r-3)(3r^2+2r-1) = 0$

if $r = 3 \rightarrow a = \frac{12}{3 \times 4} = 1 \rightarrow \boxed{1, 3, 9, 27}$

if $r = \frac{1}{3} \rightarrow a = \frac{12}{\frac{1}{3} \times \frac{4}{3}} = 27 \rightarrow \boxed{1, 3, 9, 27}$

if $r = -1 \rightarrow \times$ $\Big\} \Rightarrow \boxed{27}$ ✓

$a_1 + a_1 q + a_1 q^2 = 13 \rightarrow a_1 + a_1 q^2 = 10$

$a_1^3 q^3 = 27 \xrightarrow{\sqrt[3]{}} a_1 q = 3 = a_2$

$\rightarrow \frac{3}{q} + 3 + 3q = 13 \Rightarrow \frac{1}{q} + 1 + q = \frac{13}{3} \Rightarrow$

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$q + \frac{1}{q} = \frac{10}{3}$, $3q^2 - 10q + 3 = 0$, $q = 3$ یا $q = \frac{1}{3}$

if $q = 3 \rightarrow a = \frac{3}{q} = 1 \rightarrow 1, 3, 9$

if $q = \frac{1}{3} \rightarrow a = \frac{3}{\frac{1}{3}} = 9 \rightarrow 1, 3, 9$

$\boxed{1, 3, 9}$ ✓

الف)

$a_1 = 2$, $a_1 q = 6$, $a_1 q^2 = 18$ $\Big\}$ $q = 3$, $S_{10} = \cancel{2}\left(\frac{3^{10} - 1}{\cancel{3-1}}\right) \Rightarrow \boxed{3^{10} - 1}$ ✓

ب)

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$a_1^{10} q^{45} = ? \rightarrow \boxed{2^{10} \times 3^{45}}$ ✓

$a, aq, aq^2, aq^3 \ldots \rightarrow b_1 = aq - a = a(q-1)$

$b_2 = aq^2 - aq = aq(q-1)$

$b_3 = aq^3 - aq^2 = aq^2(q-1)$

۲

$b_n = aq^{n-1}(q-1)$ $(n \geq 1)$

$\frac{b_{n+1}}{b_n} = \frac{aq^n(q-1)}{aq^{n-1}(q-1)} = q$ ✓

الف)

$a, a+d, a+2d, \ldots, a+(n-1)d$

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$S_n = a + (a+d) + (a+2d) + \cdots + [a+(n-1)d]$ ✓

$2S_n = n[2a+(n-1)d] \rightarrow S_n = \frac{n}{2}[2a+(n-1)d]$

ب)

$a, aq, aq^2, \ldots, aq^{n-1}$, $S_n = a + aq + aq^2 + \cdots + aq^{n-1}$, $qS_n = aq + aq^2 + \cdots + aq^n$

$S_n - qS_n = a - aq^n \rightarrow S_n(1-q) = a(1-q^n) \xrightarrow{q \neq 1} S_n = \frac{a(1-q^n)}{1-q}$ یا $S_n = \frac{a(q^n-1)}{q-1}$ ✓

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