

$$q = \frac{a_2}{a_1} = 2 \Rightarrow a_n = a_1 \times q^{n-1} \quad | \quad a_n = \frac{1}{2} \times 2^{n-1} = 2^V \Rightarrow$$

$$\text{الف)} a_{10} = a_1 \times q^9 = \frac{1}{2} \times 2^9 = 2^8 = 256 \quad | \quad 2^{n-2} = 2^V \Rightarrow V = n-2 \Rightarrow \boxed{n=9} \rightarrow \text{جواب}$$

$$\text{ب)} \frac{a_{14}}{a_{12}} = \frac{a_1 \times q^{14}}{a_1 \times q^{12}} = q^2 \Rightarrow \frac{a_{14}}{a_{12}} = 2^2 = 4$$

$$\text{ج)} a_{10} \times a_{12} = (a_n)^2 \Rightarrow a_n = \sqrt{a_{10} \times a_{12}} = \sqrt{256 \times 4} = \sqrt{1024} = 32 = 2^5 \Rightarrow \boxed{a_7 = 32}$$

$$a_5 = 12 \text{ و } a_8 = 99$$

$$\frac{a_8}{a_5} = q^3 \Rightarrow a_8 = a_5 \times q^3 \Rightarrow 99 = 12q^3 \Rightarrow q^3 = \frac{99}{12} = \frac{33}{4} \Rightarrow \boxed{q=2}$$

$$a_5 = a_1 \times q^4 = 12 \Rightarrow a_1 \times 2^4 = 12 \Rightarrow a_1 = \frac{12}{16} = \frac{3}{4}$$

$$\Rightarrow a_{10} = a_1 \times q^9 = \frac{3}{4} \times 2^9 = \frac{3 \times 512}{4} = 384$$

$$a_1 \times a_2 \times a_3 \times a_4 \times a_5 = 144 \Rightarrow a_1 \times a_5 = a_2 \times a_4 = (a_3)^2 \Rightarrow$$

$$\underbrace{a_1 \times a_5}_{a_1 \times a_5} \times \underbrace{a_2 \times a_4}_{a_2 \times a_4} \times a_3 = 144 \Rightarrow (a_3)^5 = 144 = 2^4 \times 3^2 \Rightarrow \boxed{a_3 = 3}$$

$$\text{الف)} \boxed{a_3 = 3} \quad \text{ب)} a_1 \times a_5 = (a_3)^2 = 3^2 = 9 \Rightarrow a_1 \times a_5 = 9 \xrightarrow{k}$$

$$a_1 \times a_1 \times q^4 = (a_1 \times q^4)^2 \Rightarrow a_1 \times q^4 = a_3 \Rightarrow (a_1 \times q^4)^2 = 9$$

$$2^a \times 2^b = (2^{\sqrt{2}})^2 \Rightarrow 2^{\sqrt{2}} = 2^{\frac{1}{2}} \times 2^{\frac{1}{2}} = 2^{\frac{1}{2}} \times 2^{\frac{1}{2}} \Rightarrow 2^{\sqrt{2}} = \sqrt{2 \times 2} = \sqrt{2^2} = 2^1$$

$$\Rightarrow 2^a \times 2^b = (2^{\sqrt{2}})^2 = (2^{\frac{1}{2}})^2 = 2^1 = 2^a \Rightarrow a+b = 1 \Rightarrow$$

$$\boxed{a+b=1} \Rightarrow \text{رابطه صواب بین دو عدد} = \frac{a+b}{2} \Rightarrow \frac{1}{2} = \text{رابطه صواب بین دو عدد}$$

$$\text{رواق: } a_1 = a, a_2 = 0 \text{ و } a_3 = b \Rightarrow a_1 + a_3 = 2a_2 \Rightarrow 1 = 2 \times 0 \Rightarrow \boxed{a_2 = \frac{1}{2}}$$

$$a_3 = (1-x) \text{ و } a_5 = x \text{ و } a_{11} = (1+x) \Rightarrow$$

$$a_3 \times a_{11} = (a_5)^2 \Rightarrow (1+x)(1-x) = x^2 \Rightarrow x^2 = 1-x^2 \Rightarrow 2x^2 = 1 \Rightarrow x^2 = \frac{1}{2}$$

$$\Rightarrow x = \pm \sqrt{\frac{1}{2}} = \boxed{\pm \frac{\sqrt{2}}{2}}$$

$$a_1 + a_1 q^r = 21 = a_1 + a_r \Rightarrow \frac{a_1 + a_r}{a_r + a_r} = \frac{21}{12} = \frac{7}{4} \Rightarrow$$

$$\frac{a_1(q^r+1)}{a_1q(q+1)} = \frac{(q^r+1)}{(q+1)q} = \frac{(q+1)(q^r-q+1)}{q(q+1)} = \frac{7}{4} \Rightarrow 4q = 3q^r - 3q + 7 \Rightarrow$$

$$3q^r - 4q + 7 - 3q = 0 \Rightarrow 3q^r - 10q + 7 = 0 \Rightarrow (q-1)(q-1) = 0 \Rightarrow q = 1 \text{ or } 7 \Rightarrow \text{if } q = \frac{1}{7} \Rightarrow a_1 = 21$$

$$a_1 + a_r + a_r = 13 \Rightarrow a_1 + a_1q + a_1q^r = 13 \Rightarrow a_1(1+q+q^r) = 13$$

$$a_1 \times a_r \times a_r = 27 \Rightarrow a_1 \times a_1q \times a_1q^r = 27 \Rightarrow a_1^3 \times q^r = 27 \Rightarrow a_1 \times q = 3 = a_r \Rightarrow$$

$$a_1 + a_r = 13 - 3 = 10 \Rightarrow a_1 + a_1q^r = 10 \Rightarrow a_1(1+q^r) = 10 \Rightarrow a_1 = \frac{10}{(1+q^r)} \Rightarrow$$

$$a_1(1+q+q^r) = 13 \Rightarrow \frac{10(1+q+q^r)}{(1+q^r)} = 13 \Rightarrow 10q^r + 10q + 10 = 13q^r + 13 \Rightarrow 3q^r - 10q + 3 = 0$$

$$\Rightarrow q^r - 10q + 3 = 0 \Rightarrow (q-4)(q-1) = 0 \Rightarrow q = \frac{3}{4} \text{ or } 1$$

$$\text{if } q = 3 \Rightarrow a_1 \times a_1q \times a_1q^r = 27 \Rightarrow a_1^3 \times 3^r = 27 \Rightarrow a_1^3 = 1 \Rightarrow a_1 = 1 \text{ and } a_r = 3 \text{ and } a_r = 9$$

$$\text{if } q = \frac{1}{3} \Rightarrow a_1 \times a_1q \times a_1q^r = 27 \Rightarrow a_1^3 \times \left(\frac{1}{3}\right)^r = 27 \Rightarrow a_1^3 = 81 \Rightarrow a_1 = 9 \text{ and } a_r = 1 \text{ and } a_r = 1$$

(الف: جواب سوال ۱) $S_{10} = \frac{a_1(q^{10}-1)}{q-1} \Rightarrow a_1 = 2 \text{ and } q = \frac{a_r}{a_1} = \frac{4}{2} = 2 \Rightarrow$

$$S_{10} = \frac{2 \times (2^{10}-1)}{(2-1)} = (2^{10}-1) \Rightarrow \text{جواب نهایی}$$

$$a_{10} = a_1 \times q^9 = 2 \times 2^9$$

ب) $P = \left(\frac{\text{اولی} \times \text{آخری}}{r} \right)^{\frac{n}{2}} \Rightarrow (a_{10} \times a_1)^{\frac{5}{2}} \Rightarrow P = (2 \times 2^9 \times 2)^{\frac{5}{2}} = (2^2 \times 2^9)^{\frac{5}{2}} \Rightarrow$

$$P = 2^{10} \times 2^{45} \Rightarrow \text{جواب نهایی}$$

$$a_1, a_1q, a_1q^2, a_1q^3, \dots \Rightarrow$$

$$a_1q - a_1 = a_1(q-1) \text{ و } a_1q^2 - a_1q = a_1q(q-1) \text{ و } a_1q^3 - a_1q^2 = a_1q^2(q-1)$$

$$\Rightarrow \text{دنباله جدید} = q' \Rightarrow a_1(q-1), a_1q(q-1) \text{ و } a_1q^2(q-1) \text{ و } \dots$$

$$q' = \frac{a_r}{a_1} = \frac{a_1q^n}{a_1(q-1)} = q \Rightarrow q' = q \Rightarrow \text{دنباله قدیم جدیدی نمی دهد}$$

(الف) $a_1 + a_2 + a_3 + \dots + a_n = a_1 + (a_1+d) + (a_1+2d) + \dots + (a_1+(n-1)d)$

$$\Rightarrow a_1 + (a_1+d) + (a_1+2d) + \dots + (a_1+(n-1)d) = na_1 + d(1+2+\dots+(n-1))$$

$$1+2+3+\dots+(n-1) = \frac{(n-1)(n-1+1)}{2} = \frac{n(n-1)}{2} \Rightarrow S_n = na_1 + \left(\frac{n(n-1)}{2}\right)d \Rightarrow$$

$$S_n = na_1 + d \times \frac{n}{2} \times (n-1) = \frac{n}{2} (2a_1 + (n-1)d) \Rightarrow S_n = a_1 + a_1q + a_1q^2 + \dots + a_1q^{n-1}$$

$$S_n = a_1(1+q+q^2+\dots+q^{n-1}) \Rightarrow qS_n - S_n = a_1(q+q^2+q^3+\dots+q^n) - (1+q+q^2+\dots+q^{n-1})$$

$$\Rightarrow (q-1)S_n = a_1(q^n-1) \Rightarrow S_n = a_1 \frac{(q^n-1)}{(q-1)}$$