

$q = \frac{a_2}{a_1} = 2 \Rightarrow a_n = a_1 \times q^{n-1}$ ۱) $a_n = \frac{1}{2} \times 2^{n-1} = 2^V \Rightarrow$
الف) $a_{10} = a_1 \times q^9 = \frac{1}{2} \times 2^9 = 2^8 = 256$ ✓ $2^{n-2} = 2^V \Rightarrow V = n-2 \Rightarrow n=9 \rightarrow$ جایزیم
ب) $\frac{a_{14}}{a_{12}} = \frac{a_1 \times q^{14}}{a_1 \times q^{12}} = q^2 \Rightarrow \frac{a_{14}}{a_{12}} = 2^2 = 4$ ✓ (۲)
ج) $a_{10} \times a_{12} = (a_n)^2 \Rightarrow a_n = \sqrt{a_{10} \times a_{12}} = \sqrt{256 \times 4} = \sqrt{2^{10}} = 2^5 = 32 \Rightarrow a_{11} = 32$ ✓

$a_5 = 12$ و $a_8 = 99$
 $\frac{a_8}{a_5} = q^3 \Rightarrow a_8 = a_5 \times q^3 \Rightarrow 99 = 12q^3 \Rightarrow q^3 = \frac{99}{12} = \frac{33}{4} \Rightarrow q = \frac{33}{4} = 1 \Rightarrow q = 2$ (۲)
 $a_5 = a_1 \times q^4 = 12 \Rightarrow a_1 \times 2^4 = 12 \Rightarrow a_1 = \frac{12}{16} = \frac{3}{4}$
 $\Rightarrow a_{10} = a_1 \times q^9 = \frac{3}{4} \times 2^9 = \frac{3 \times 512}{4} = 384$ ✓

$a_1 \times a_2 \times a_3 \times a_4 \times a_5 = 144 \Rightarrow a_1 \times a_5 = a_2 \times a_4 = (a_3)^2 \Rightarrow$
 $\frac{a_2 \times a_4}{a_1 \times a_5} \times \frac{a_3 \times a_3}{a_2 \times a_4} = 144 \Rightarrow (a_3)^5 = 144 = 2^4 \times 3^2 \Rightarrow a_3 = 3$ (۲)
الف) $a_3 = 3$ ب) $a_1 \times a_5 = (a_3)^2 = 3^2 = 9 \Rightarrow a_1 \times a_5 = 9$ $\xrightarrow{k}$
 $a_1 \times a_1 \times q^4 = (a_1 \times q^4)^2 \Rightarrow a_1 \times q^4 = a_3 \Rightarrow (a_1 \times q^4)^2 = 9$ ✓

$2^a \times 2^b = (2^{\sqrt{2}})^2 \Rightarrow \sqrt{2} = 2^{\frac{1}{2}} \Rightarrow \sqrt{2} = \sqrt{2 \times 2} = \sqrt{2^2} \Rightarrow$
 $\Rightarrow 2^a \times 2^b = (\sqrt{2^2})^2 = (2^1)^2 = 2^2 = 2^a \Rightarrow a+b = 2 \Rightarrow$
 $a+b=2 \Rightarrow$ (۲)
 $\Rightarrow \frac{a+b}{2} = 1$ ۱، ۱
 روابط: $a_1 = a$ و $a_2 = 0$ و $a_3 = b \Rightarrow a_1 + a_3 = 2a_2 \Rightarrow a+b=0 \Rightarrow a_2 = \frac{a+b}{2}$ ✓

$a_3 = (1-x)$ و $a_4 = x$ و $a_{11} = (1+x) \Rightarrow$ ۱، ۱، ۵
 $a_3 \times a_{11} = (a_4)^2 \Rightarrow (1+x)(1-x) = x^2 \Rightarrow 1-x^2 = x^2 \Rightarrow 2x^2 = 1 \Rightarrow x^2 = \frac{1}{2} \Rightarrow x = \pm \sqrt{\frac{1}{2}}$
 $\Rightarrow x = \pm \sqrt{\frac{1}{2}} = \pm \frac{\sqrt{2}}{2}$ فقط حالت + قابل قبول است زیرا به ازای
۲ جداول فرد با هم غیر هم علامت می‌باشد

$$a_1 + a_1 q^r = 21 = a_1 + a_1 r \Rightarrow \frac{a_1 + a_1 r}{a_1 + a_1 r} = \frac{21}{11} = \frac{V}{r} \Rightarrow$$

$$\frac{a_1(q^r+1)}{a_1q(q+1)} = \frac{(q^r+1)}{(q+1)q} = \frac{(q+1)(q^r-q+1)}{q(q+1)} = \frac{V}{r} \Rightarrow Vq = r q^r - r q + r \Rightarrow$$

$$r q^r - Vq + r - r q = 0 \Rightarrow r q^r - 10q + r = 0 \Rightarrow (q-1)(q-1) = 0 \Rightarrow q = \frac{1}{r} \text{ و } r = \frac{1}{q} \Rightarrow \text{if } q = \frac{1}{r} \Rightarrow a_1 = 2V$$

$$a_1 + a_1 r + a_1 r^2 = 13 \Rightarrow a_1 + a_1 q + a_1 q^r = 13 \Rightarrow a_1(1 + q + q^r) = 13$$

$$a_1 \times a_1 r \times a_1 r^2 = 2V \Rightarrow a_1 \times a_1 q \times a_1 q^r = 2V \Rightarrow a_1^3 \times q^r = 2V \Rightarrow a_1 \times q = r = a_1 \Rightarrow$$

$$a_1 + a_1 r = 13 - 2 = 11 \Rightarrow a_1 + a_1 q^r = 11 \Rightarrow a_1(1 + q^r) = 11 \Rightarrow a_1 = \frac{11}{(1+q^r)} \Rightarrow$$

$$a_1(1 + q + q^r) = 13 \Rightarrow \frac{11(1 + q + q^r)}{(1 + q^r)} = 13 \Rightarrow 11q^r + 11q + 11 = 13q^r + 13 \Rightarrow r q^r - 10q + r = 0$$

$$\Rightarrow q^r - 10q + q = 0 \Rightarrow (q-4)(q-1) = 0 \Rightarrow q = \frac{q}{r} = 3 \text{ و } \frac{1}{r} = 3 \Rightarrow r = \frac{1}{3}$$

$$\text{if } q = 3 \Rightarrow a_1 \times a_1 q \times a_1 q^r = 2V \Rightarrow a_1^3 \times 3^r = 2V \Rightarrow a_1^3 = 1 \Rightarrow a_1 = 1 \text{ و } a_1 r = 3 \text{ و } a_1 r^2 = 9$$

$$\text{if } q = \frac{1}{3} \Rightarrow a_1 \times a_1 q \times a_1 q^r = 2V \Rightarrow a_1^3 \times \left(\frac{1}{3}\right)^r = 2V \Rightarrow a_1^3 = 2V \Rightarrow a_1 = 9 \text{ و } a_1 r = 1 \text{ و } a_1 r^2 = 1$$

$$(الف) \text{ جواب سوال } S_{10} = \frac{a_1(q^{10}-1)}{q-1} \Rightarrow a_1 = 2, q = \frac{a_1 r}{a_1} = \frac{4}{2} = 2 \Rightarrow$$

$$S_{10} = \frac{2 \times (2^{10}-1)}{(2-1)} = (2^{10}-1) \Rightarrow \text{جواب نهایی}$$

$$P = (a_1 \times a_1 r \times a_1 r^2 \times \dots \times a_1 r^{n-1})^{\frac{1}{n}} \Rightarrow (a_{10} \times a_1)^{\frac{1}{n}} \Rightarrow P = (2 \times 2^9)^{\frac{1}{10}} = (2^{10})^{\frac{1}{10}} = 2$$

$$a_1, a_1 q, a_1 q^2, a_1 q^3, \dots \Rightarrow$$

$$a_1 q - a_1 = a_1(q-1) \text{ و } a_1 q^2 - a_1 q = a_1 q(q-1) \text{ و } a_1 q^3 - a_1 q^2 = a_1 q^2(q-1)$$

$$\Rightarrow \text{دنباله جدید} = q' \Rightarrow a_1(q-1), a_1 q(q-1) \text{ و } a_1 q^2(q-1) \text{ و } \dots$$

$$q' = \frac{a_1 r}{a_1} = \frac{a_1 q(q-1)}{a_1(q-1)} = q \Rightarrow q' = q \Rightarrow \text{دنباله قدیم جدیدی نمی دهد}$$

$$(الف) \text{ فرض: } a_1 + a_1 r + a_1 r^2 + \dots + a_1 r^{n-1} = a_1 + (a_1 + d) + (a_1 + 2d) + \dots + (a_1 + (n-1)d)$$

$$\Rightarrow a_1 + (a_1 + d) + (a_1 + 2d) + \dots + (a_1 + (n-1)d) = n a_1 + d(1 + 2 + \dots + (n-1))$$

$$1 + 2 + 3 + \dots + (n-1) = \frac{(n-1)(n-1+1)}{2} = \frac{n(n-1)}{2} \Rightarrow S_n = n a_1 + \left(\frac{n(n-1)}{2}\right) d$$

$$S_n = n a_1 + d \times \frac{n}{2} \times (n-1) = \frac{n}{2} (2a_1 + (n-1)d) \Rightarrow S_n = a_1 + a_1 q + a_1 q^2 + \dots + a_1 q^{n-1}$$

$$S_n = a_1(1 + q + q^2 + \dots + q^{n-1}) \Rightarrow q S_n - S_n = a_1(q + q^2 + q^3 + \dots + q^n) - (1 + q + q^2 + \dots + q^{n-1})$$

$$\Rightarrow (q-1)S_n = a_1(q^n - 1) \Rightarrow S_n = a_1 \frac{(q^n - 1)}{(q-1)}$$