کلاس دهم B

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$a_n = \frac{1}{2} , 1 , 2 , \dots$

الف) $a_{10} = \frac{1}{2} \times 2^9 = 256$

ب) $\frac{a_{17}}{a_{13}} = q^4 = 2^4 = 16$

ج) $4 , 8 , 16 , 32 , 64 , 128 , 256$

د) $128 = \frac{1}{2} \times 2^n \Rightarrow n = 8$, $8+1=9$, a_9

۲-

$a_8 = 96$

$a_5 = 12$

$\frac{a_8}{a_5} = q^3 = \frac{96}{12} = 8 \Rightarrow q = 2$

$a_{10} = a_8 q^2 = 96 \times 4 = 384$

۳-

$a_1 \times a_2 \times a_3 \times a_4 \times a_5 = 243$

$a_1 \times a_5 = a_2 \times a_4 = a_3 \times a_3$

الف) $\Rightarrow a_3 \times a_3 \times a_3 \times a_3 \times a_3 = 243 = a_3^5 \Rightarrow a_3 = 3$

ب) $a_1 \times a_5 = a_3 \times a_3 = 3 \times 3 = 9$

۴-

$2^b , 4\sqrt{2} , 2^a$

$(4\sqrt{2})^2 = 2^{a+b} = 32 \Rightarrow 2^{a+b} = 2^5 \Rightarrow a+b = 5$

$\frac{a+b}{2} =$ ۲/۵

۵-

$a_3 = 1 - x$

$a_7 = x$

$a_{11} = x + 1$

$\frac{a_7}{a_3} = \frac{a_{11}}{a_7} = q^4 \Rightarrow \frac{x}{1-x} = \frac{x+1}{x}$

$\Rightarrow x^2 = \cancel{x} + 1 - x^2 - \cancel{x} \Rightarrow 2x^2 = 1 \Rightarrow x^2 = \frac{1}{2}$

$\Rightarrow x = \pm\frac{1}{\sqrt{2}} \Rightarrow x = \pm\frac{1}{\sqrt{2}}$

۶-

$$a_1 + a_4 = 28 \Rightarrow \frac{a_1 + a_1 q^3}{a_1 q + a q^2} = \frac{\cancel{a_1}(1+q^3) = 28}{\cancel{a_1}(q+q^2) = 12}$$
$$a_2 + a_3 = 12$$

$\Rightarrow q = 3 \;,\; a_1 = 1$ بزرگترین $= a_4 = 1 \times 27 = 27$

۷-

$$a_1 + a_2 + a_3 = 13 = a_1 + a_1 q + a_1 q^2$$
$$a_1 \times a_2 \times a_3 = 27 = a_1^3 q^3$$
$$a_1 \times a_3 = a_2^2 \Rightarrow a_2 \times a_2 \times a_2 = 27 \Rightarrow \boxed{a_2 = 3 \;,\; a_1 = 1 \;,\; a_3 = 9}$$

۸-

$$a_n = 2, 6, 18, \dots \Rightarrow q = 3 \qquad \left(\frac{q^n - 1}{q - 1}\right) a_1 = S_n = \cancel{2}\left(\frac{3^{10} - 1}{\cancel{3-1}}\right) = 3^{10} - 1$$

الف)

$$\boxed{3^{10} - 1 = 59048}$$

ب)

$$P_n = (a_1 \times L)^{\frac{n}{2}} \Rightarrow P_{10} = (2 \times 39366)^5 = \boxed{78732^5}$$

$$a_n = a_1 q^{n-1} \Rightarrow a_{10} = 2 \times 3^9 = 39366$$

۹-

$$a_n = a, aq, aq^2, aq^3, \dots$$
$$a - aq = a(1-q)$$
$$aq - aq^2 = aq(1-q)$$
$$aq^2 - aq^3 = aq^2(1-q)$$

$$\boxed{a_n = a(1-q) \xrightarrow{q} aq(1-q) \xrightarrow{q} aq^2(1-q)}$$

$$\boxed{q' = q}$$

۱۰- الف)

$$S_n = a_1 + a_2 + a_3 + \dots + a_n = a_1 + a_1 + d + a_1 + 2d + \dots + a_1 + (n-1)d$$
$$= n a_1 + \left(\frac{n^2 - n}{2}\right) d \xrightarrow{\text{فاکتور}} \frac{n}{2}\left(2a_1 + (n-1)d\right)$$

ب)

$$\left(S = a + aq + aq^2 + \dots + aq^{n-1}\right) \times q \Rightarrow Sq = aq + aq^2 + \dots + aq^n$$

$$Sq - S = \left(\cancel{aq} + \cancel{aq^2} + \dots + \cancel{aq^{n-1}} + aq^n\right) - \left(a + \cancel{aq} + \cancel{aq^2} + \dots + \cancel{aq^{n-1}}\right)$$

$$\Rightarrow Sq - S = aq^n - a \Rightarrow S(q-1) = a(q^n - 1) \Rightarrow S = a\frac{(q^n - 1)}{q - 1}$$

$$q \neq 1$$