تکلیف ۱۴ — به نام خدا — ایلیا رضایی دهم B

(۱)

$a_n = \frac{1}{2}, 1, 2, \dots$

الف) $a_n = \frac{1}{2} \times 2^{(n-1)} \rightarrow \frac{1}{2} \times 2^9 = 2^8 = \boxed{256}$

ب) $\frac{a_{17}}{a_{12}} = \frac{aq^{16}}{aq^{12}} = q^4 \rightarrow 2^4 = \boxed{16}$

ج) $b^2 = ac \rightarrow b^2 = a_4 \times a_{10} = 2^2 \cdot 2^8 = 2^{10} \rightarrow b^2 = 2^{10}$

$a_4 = \frac{1}{2} 2^3 = 2^2 \quad a_{10} = \frac{1}{2} 2^9 = 2^8$

$\rightarrow b = \boxed{2^5} = \boxed{32}$

د) $128 = \frac{1}{2} \times 2^{(n-1)} \rightarrow 256 = 2^{(n-1)} \rightarrow 2^8 = 2^{n-1} \rightarrow n = \boxed{9}$

(۲)

$q^{m-n} = \frac{a_m}{a_n} \rightarrow \frac{a_1}{a_5} = q^3 \rightarrow q^3 = \frac{96}{16} \rightarrow q^3 = 2^3 \Rightarrow q = 2$

$a_{18} \times q^2 = a_{10} \rightarrow a_{10} = 96 \times 2^2 = \boxed{384}$

(۳)

$P_n = (\text{مجموع})^n$

الف) $a_1 \times a_2 \times a_3 \times a_4 \times a_5 = 243 \rightarrow a_3^5 = 243 \rightarrow a_3 = \boxed{3}$

ب) $a_1 \times a_5 = a_4 \times a_2 \rightarrow a_1 \times a_5 = \boxed{9}$

۴) $2^a, 4\sqrt{2}, 2^b$

$b^2 = ac \rightarrow (4\sqrt{2})^2 = (\sqrt{32})^2 = 2^{a+b} \rightarrow 2^5 = 2^{a+b} \rightarrow a+b=5$

$a, x, b \rightarrow x = \frac{a+b}{2} \rightarrow x = \boxed{2.5}$

۵) $a_3 = aq^2 = 1-x$

$a_7 = aq^6 = x$ $\quad a_3 \times a_{11} = a_7 \times a_7$

$a_{11} = aq^{10} = x+1$ $\quad \rightarrow (1-x)(1+x) = x^2$

$\rightarrow 1-x^2 = x^2 \rightarrow 2x^2 = 1$

$\rightarrow x^2 = \frac{1}{2} \begin{cases} x = \boxed{\frac{1}{\sqrt{2}}} \\ x = \boxed{-\frac{1}{\sqrt{2}}} \end{cases}$

$x = \pm \boxed{\frac{\sqrt{2}}{2}}$

۶) $a_1 + a_5 = a + aq^3 = 28 \rightarrow a(1+q^3) = 28$

$a_2 + a_3 = aq + aq^2 = 12 \rightarrow aq(1+q) = 12$

$\frac{a_1+a_5}{a_2+a_3} = \frac{28}{12} \rightarrow \frac{a(1+q^3)}{aq(1+q)} = \frac{7}{3} \rightarrow \frac{(q+1)(q^2-q+1)}{q(1+q)} = \frac{7}{3}$

$\rightarrow 3q^2 - 3q + 3 = 7q \rightarrow 3q^2 - 10q + 3 = 0$

روش $\rightarrow q^2 - 10q + 9 = 0 \rightarrow (q-9)(q-1) \rightarrow q \begin{cases} q = \frac{9}{3} = 3 \\ q = \frac{1}{3} \end{cases}$

$\rightarrow a_1 + a_5 = 28 \rightarrow a_1 + a_1 q^4 = 28 \rightarrow \begin{cases} a_1 = 1 \\ a_1 = 27 \end{cases}$

$1, 3, 9, \boxed{27}$ یا $\boxed{27}, 9, 3, 1$

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۷) $a \cdot aq \cdot aq^2 = a^3q^3 = 27 \Rightarrow aq = 3 \rightarrow a = \frac{3}{q}$

$a + aq + aq^2 = 13 \longrightarrow a(1 + q + q^2) = 13$

$\longrightarrow \frac{3}{q}(1 + q + q^2) = 13 \rightarrow 3(1 + q + q^2) = 13q$

$\longrightarrow 3q^2 + 3q + 3 = 13q \rightarrow 3q^2 - 10q + 3 = 0$

$\longrightarrow q^2 - 10q + 9 = 0 \longrightarrow (q-1)(q-9)$

$\longrightarrow \begin{cases} q = \frac{1}{3} \\ q = \frac{9}{3} = 3 \end{cases}$

$\longrightarrow$ 1, 3, 9 یا 9, 3, 1

۱۰) $a_n = 2, 6, 18, \ldots \quad a_n = aq^{n-1} = 2 \times 3^{n-1}$

(×3 ، ×3)

الف) $S_n = a\left(\frac{q^n - 1}{q - 1}\right) \rightarrow S_{10} = 2\frac{3^{10} - 1}{3 - 1} = \boxed{3^{10} - 1}$

ب) $P_n = (a_1 \times a_n)^{\frac{n}{2}} \rightarrow P_{10} = (2 \times 2 \times 3^9)^5 = \boxed{2^{10} \times 3^{45}}$

$a_{10} = 2 \times 3^9$

۹) $a - aq \;,\; aq - aq^2 \;,\; aq^2 - aq^3$

$(aq - aq^2)^2 = (a - aq)(aq^2 - aq^3)$

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$(aq - aq^2)^2 = (a - aq)(aq^2 - aq^3)$

$\rightarrow a^2(q - q^2)^2 = a^2(1-q)(q^2 - q^3)$

$\rightarrow q^2 + q^4 - 2q^3 = q^2 - q^3 - q^3 + q^4$

$\rightarrow q^2 + q^4 - 2q^3 = q^2 + q^4 - 2q^3$ ✓

$a - aq \; , \; aq - aq^2 \; , \; aq^2 - aq^3$

$\rightarrow a(1-q) \; , \; a(q - q^2) \; , \; a(q^2 - q^3)$

$\times q \qquad \times q$

$q = q$ ثابت

۱۲

الف)

$S_n = \frac{n}{2}(2a + (n-1)d)$

$a_1 \; , \; a_2 \; , \; a_3 \; , \; \dots \; , \; a_{n-2} \; , \; a_{n-1} \; , \; a_n$

$a_1 + a_n = a_2 + a_{n-1} = a_3 + a_{n-2}$

$S_n = (a_1 + a_n)\frac{n}{2} \xrightarrow{a_n = a_1 + (n-1)d} S_n = \frac{n}{2}(a_1 + a_1 + (n-1)d)$

$\rightarrow S_n = \frac{n}{2}(2a_1 + (n-1)d)$

$$S_n = a\left(\frac{q^n - 1}{q - 1}\right)$$

$$a, aq, aq^2, aq^3, \ldots, aq^{(n-1)}$$

$$S_n = a + aq + aq^2 + aq^3 + \ldots + aq^{(n-1)}$$

$\times q$

$$qS_n = aq + aq^2 + aq^3 + \ldots + aq^n$$

$$S_n - qS_n = a - aq^n \rightarrow S_n(1-q) = a(1-q^n)$$

$$\rightarrow S_n = \frac{a(1-q^n)}{(1-q)} = S_n = a\left(\frac{q^n - 1}{q-1}\right)$$