

الف) $a_n = aq^{n-1} \rightarrow a_{10} = \frac{1}{r} \times r^9 = r^8 = 128$

ب) $\frac{a_{14}}{a_{13}} = \frac{aq^{14}}{aq^{13}} = q = r^8 = 16$

2) $\sqrt{a_4 \times a_{10}} = \sqrt{a_1 r^3 \times a_1 r^9} = \sqrt{a_1^2 r^{12}} = a_1 r^6 = \frac{1}{r} \times r^6 = r^5 = 32$

ج) $\frac{1}{r} \times r^{n-1} = 128 \rightarrow r^{n-1} = 128 \rightarrow r^8 = 128 \rightarrow n-1=8 \rightarrow n=9$
 $\rightarrow a_9 = 128$

$\left. \begin{array}{l} a_5 = aq^4 = 12 \\ a_1 = aq^0 = 96 \end{array} \right\} \rightarrow q^4 = 8 \rightarrow q = 2$

$a_{10} = a_1 \times q^9 = 96 \times 2^9 = 12288$

الف) $a_1 \times a_2 \times a_3 \times a_4 \times a_5 = 243 \rightarrow a_1 r \times a_1 r^2 \times a_1 r^3 \times a_1 r^4 \times a_1 r^5 = 243$
 $\rightarrow a_1^5 r^{15} = 243 \rightarrow a_1^5 = 243 \rightarrow a_1 = 3$

$\Rightarrow a_1 \times a_5 = a_3^2 \rightarrow \left\{ \begin{array}{l} a_5 = 3^5 \\ a_1 \times a_5 = a_3^2 \end{array} \right\} \rightarrow a_1 \times a_5 = 3^5 = 243$
 $\rightarrow a_1 \times a_5 = 243$

$r^a \times r^b = (r^{\sqrt{r}})^r \rightarrow r^{a+b} = r^r \rightarrow r^{a+b} = r^r \rightarrow a+b=r$

واسطه a و b $= \frac{a+b}{2} = \frac{r}{2} = 12$
 $\rightarrow r = 24$

$(a_7)^r = a_1 \times a_{11} \rightarrow x^r = (1-x)(1+x) \rightarrow x^r = 1-x^2 \rightarrow 2x^r = 1$

$\rightarrow x^r = \frac{1}{2} \rightarrow x = \pm \frac{1}{\sqrt{2}} = \pm \frac{\sqrt{2}}{2}$

مقدار x می شود $\frac{-\sqrt{2}}{2}$ و $\frac{\sqrt{2}}{2}$

$$\begin{aligned}
 a + aq^r &= 18 \rightarrow a(1+q^r) = 18 \rightarrow a = \frac{18}{1+q^r} \\
 aq + aq^r &= 12 \rightarrow aq(1+q) = 12 \rightarrow a = \frac{12}{q(1+q)}
 \end{aligned}
 \left\{ \begin{aligned}
 \frac{18}{1+q^r} &= \frac{12}{q(1+q)} \\
 \frac{18}{q(1+q)} &= 18 \Rightarrow 18(1+q^r) = 18q(1+q) \Rightarrow 18 + 18q^r = 18q + 18q^2
 \end{aligned} \right.
 \rightarrow 18q^r - 18q^2 - 18q + 18 = 0 \rightarrow 18q^r - 18q^2 - 18q + 18 = 0 \rightarrow q = 3 \rightarrow 3(18) - 18(9) - 18(3) + 18 = 0$$

$$a = \frac{12}{3(1+3)} = \frac{12}{12} = 1 \rightarrow a_1, a_2, a_3, a_4 = 1, 3, 9, 27 \rightarrow \text{بارشمار 27 است}$$

$$\begin{aligned}
 a_1 + a_2 + a_3 &= 13 \\
 a_1 \times a_2 \times a_3 &= 13
 \end{aligned}
 \left\{ \begin{aligned}
 a_1 \times a_2 &= 13 \rightarrow a_1 = 13 \\
 a_2 &= 1
 \end{aligned} \right.
 \rightarrow a_1 + a_2 + a_3 = 13 \rightarrow \frac{13}{q} + 1 + 13q = 13 \rightarrow \frac{13}{q} + 13q = 12 \rightarrow 13q^2 - 12q + 13 = 0 \rightarrow (q-1)(q-1) = 0 \rightarrow q = 1$$

$$q = \frac{1}{3} \rightarrow a_1 = 9, a_2 = 3, a_3 = 1 \rightarrow a_n = 9, 3, 1, 0.000$$

$$S_n = a_1 \left(\frac{q^n - 1}{q - 1} \right) \rightarrow S_{10} = 1 \times \left(\frac{3^{10} - 1}{3 - 1} \right) = 1 \times \frac{3^{10} - 1}{2} = 3^{10} - 1 = 59048$$

$$a_1 \times a_2 \times a_3 \times a_4 \times a_5 \times a_6 \times a_7 \times a_8 \times a_9 \times a_{10} = (a_1 \times q^9)^{10} = (1 \times 3^9)^{10} = 3^{90}$$

$$a_n = a_1 a_2 \dots a_n = a \times q^{n-1} \rightarrow b_n = a_{n+1} - a_n \rightarrow b_1 = a_2 - a_1 = aq - a = a(q-1) \rightarrow b_2 = a_3 - a_2 = aq^2 - aq = a(q-1)q \rightarrow b_n = a_1 \times q^{n-1} = a(q-1) \times q^{n-1} \rightarrow b_n = a(q-1) \times q^{n-1}$$

$$q' = \frac{b_2}{b_1} = \frac{a(q-1)q}{a(q-1)} = q \rightarrow q' = q$$

$$\begin{aligned}
 S_n &= a_1 + (a_1 + d) + (a_1 + 2d) + \dots + (a_1 + (n-1)d) \\
 S_n &= (a_1 + (n-1)d) + (a_1 + (n-2)d) + \dots + a_1
 \end{aligned}
 \left\{ \begin{aligned}
 S_n &= n(a_1 + a_n) \\
 S_n &= n(2a_1 + (n-1)d) \\
 S_n &= \frac{n}{2}(2a_1 + (n-1)d) \\
 S_n &= \frac{n}{2}(a_1 + a_n)
 \end{aligned} \right.$$

$$S_n = a + aq + aq^2 + \dots + aq^{n-1} \rightarrow q \times S_n = aq + aq^2 + aq^3 + \dots + aq^n \rightarrow S_n - (q \times S_n) = (a + aq + aq^2 + \dots + aq^{n-1}) - (aq + aq^2 + aq^3 + \dots + aq^n) \Rightarrow S_n(1-q) = a - aq^n \rightarrow S_n = a \left(\frac{1-q^n}{1-q} \right) = a \left(\frac{q^n - 1}{q - 1} \right)$$