ایلیا خاکسار — دهم B — ۲۰

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$a_n = a_1+(n-1)d \quad a_4 = a_1+3d \Rightarrow 3d+40=61 \Rightarrow d=7 \Rightarrow 7(n-1)+40=208$

$168 \div 7 = n-1 = 24 \Rightarrow \boxed{n=25}$

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$n=7k+3 \Rightarrow 7k+3 \geq 100 \Rightarrow k \geq 13.86 \qquad 101 = 3+(14)7 \Rightarrow k=14$

$7k+3 \leq 999 \Rightarrow k \leq 142.28 \qquad 997 = 3+(142)7 = 142 = k$

$142-14+1 = \boxed{129}$

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$a_n = a_1+(n-1)d \quad a_{n+1}=a_1+nd \quad a_{n+2} = a_1+(n+1)d \Rightarrow a_{n+3}=a_1+(n+2)d$

$a_{n+3}+a_{n+2}+a_{n+1} = -6n+12 \qquad 6n-12 = [a_1+nd]+[a_1+(n+1)d]+[a_1+(n+2)d]$

$3a_1+nd+(n+1)d+(n+2)d = -6n+12 \qquad 3a_1+3nd+3d = -6n+12 \Rightarrow d=-2$

$a_n = 8-2n \qquad a_{17} = 8-2(17) = -26 \qquad a_8 = 8-2(8) = -8 \qquad (-26)+(-8) = \boxed{-34}$

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$a+2d = 2^x-1 \qquad a+7d = 2^{x+1} \qquad a+12d = 2^{2x}-3$

$(a+7d)-(a+2d) = 2^{x+1}-(2^x-1) \Rightarrow a+12d = 2^{2x}-3 \Rightarrow t=2^x \Rightarrow t^2-3t-4=0$

$\Rightarrow x^2=4 \Rightarrow x=2 \quad 2^x>0 \Rightarrow S=(a+2d)+(a+7d)+(a+12d)=3a+21d=\boxed{24}$

$d = \frac{2^2+1}{5} = 1$

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$a_{10}=a+9d \qquad a_{12}-a_{10}=5 \qquad 2d=5 \Rightarrow d=2.5 \qquad a_{10}+a_{12}=25$

$(a+9d)+(a+11d)=25 \qquad 2a+20d=25 \qquad a+10d=12.5 \qquad a+25=12.5$

$a=-12.5 \qquad a_{21}=a+20d=-12.5+20(2.5)=\boxed{37.5}$

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$S_5 = \frac{5}{2}[2a+4d]$ $\quad S_{6-10} = \frac{5}{2}[2(a+5d)+4d] = \frac{5}{2}[2a+14d]$ ۶

$S_5 = \frac{1}{3}S_{6-10}$ $\quad \frac{5}{2}(2a+4d) = \frac{1}{3}\cdot\frac{5}{2}(2a+14d)$ $= 6a+12d = 2a+14d \Rightarrow d = 2a$

$\frac{a_2}{a_1} = \frac{3a}{a} =$ $\boxed{\text{برابر ۳}}$

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$t_n = a+(n-1)d$ $\quad t_{n-3} = a+(n-4)d$ $\quad a+(n-1)d = a+(n-4)d+15$ $\quad d=5$

$t_n = a+(n-1)5$ $\quad t_5 = a+20$ $\quad t_9 = a+40$ $\quad t_9^2 - t_5^2 = (a+40)^2-(a+20)^2$ ۷

$t_7 = a+30$ $\quad \frac{t_9^2-t_5^2}{t_7} = \frac{40(a+30)}{a+30} =$ $\boxed{\text{۴۰ برابر}}$

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$a_1 = 2(1-6\sqrt{3})$ $\quad a_{14} = 2+\sqrt{3}$ $\quad a_n = a_1+(n-1)d$ $\quad a_{14} = a_1+13d$

$2+\sqrt{3} = 2-12\sqrt{3}+13d$ $\quad \sqrt{3} = -12\sqrt{3}+13d$ $\quad \Rightarrow \boxed{d=\sqrt{3}}$ ۸

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$1+(4n-3)+1 = 4n-1$ $\quad a_n = a_1+(n-1)d$ $\quad 32 = 2+((4n-1)-1)d = 2+(4n-2)d$

$30 = (4n-2)d \Rightarrow d = \frac{30}{4n-2}$ $\quad \Rightarrow d = \frac{9}{n}$ $\quad n(4n-2)$ $\quad 30n = 9(4n-2)$ ۹

$\Rightarrow 36n-30n = 18 \Rightarrow 6n = 18 \Rightarrow \boxed{n=3}$

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$a_n + a_{n+4} = a_3 + a_{17} \Rightarrow a+(n-1)d + a+(n+3)d = a+2d+a+16d$

$\Rightarrow 2a+(2n+2)d$ $\quad 2a+18d$ $\quad 2a+(2n+2)d = 2a+18d \Rightarrow 2n+2 = 18$ ۱۰

$\boxed{n=8}$ $\quad a_n = 3\times a_{\frac{n}{2}} \Rightarrow a_8 = 3a_4$ $\quad a_8 = a+7d$ $\quad a_4 = a+3d$

$a_k = a+(k-1)d = 0 \Rightarrow (k-2)d = 0$ $\quad k-2 = 0 \Rightarrow \boxed{k=2}$

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