

الف) $t_n = 5 + (n-1)4 = 4n + 1$ ✓

ب) $t_{10} = (4 \times 10) + 1 = 41$ ✓

(۲)

الف) $S_n = \frac{n}{2} (2a_1 + (n-1)d) \Rightarrow S_{10} = 5(10 + 9 \times 4) = 5 \times 46 = 230$ ✓

ب) $t_{19} = 4 + (18 \times 4) = 76 \Rightarrow$

$t_{19} + t_{20} + t_{21} + t_{22} = \frac{4}{2} \left((2 \times 76) + \frac{(3 \times 4)}{12} \right) = 2 \times 76 = 152$ ✓

(۲)

$1 + \sqrt{3}, 2, 3 - \sqrt{3}, \dots$
 $\begin{matrix} \nearrow & \nwarrow \\ +1-\sqrt{3} & +1-\sqrt{3} \end{matrix}$

$d = 1 - \sqrt{3}$

$t_{10} = t_{19} + 10d \Rightarrow$

$t_{10} - t_{19} = 10d = 10(1 - \sqrt{3}) = 10 - 10\sqrt{3}$ ✓

(۲)

$a_n = 0, 3 \times a, a^2 \Rightarrow a_1 = 0 \times a = 0 \Rightarrow 2a + 1 = 2 \Rightarrow 2a - 2 = -1$ ①

$b_n = x, 2, y \Rightarrow x + y = 2 \times 2 = 4$ ②

①, ② $\begin{cases} 2x - y = -1 \\ x + y = 4 \end{cases} \Rightarrow \begin{cases} x = 1 \\ y = 3 \end{cases}$ ✓

$\Rightarrow xy = 1 \times 3 = 3$ ✓

$2x - 2, 2x - 1, 2x, \dots \Rightarrow 2(2x - 1) = 2x - 2 + 2x \Rightarrow 2x - 1 = 2x - 2 \Rightarrow x = \frac{1}{2}$

$\Rightarrow -3, 0, 3, \dots \Rightarrow a_1 = 0$ ✓

(۲)

$$a_n = 2, 5, 7, \dots \Rightarrow a_7 = 2 + (19 \times 2) = 41$$

$$b_n = 2, 5, 8, \dots \Rightarrow b_7 = 2 + (19 \times 3) = 59$$

$$c_n = 5, 11, 17, \dots \Rightarrow 5 + (n-1) \times 6 \leq 41 \Rightarrow$$

$$n-1 \leq \frac{41-5}{6} \Rightarrow n \leq 7 \Rightarrow \text{7 جمله مشترک دارند}$$

$$a_1 + a_r + a_w = 21 \Rightarrow 3a_r = 21 \Rightarrow a_r = 7$$

$$a_1 + a_r + a_w = 105 \Rightarrow 3a_w = 105 \Rightarrow a_w = 35$$

$$\frac{a_w - a_r + a_1}{a_1} = \frac{35 - 7 - 21}{-21} = -\frac{7}{21} = -\frac{1}{3}$$

$$a_1 + a_r + a_w = 105 \Rightarrow 3a_r = 105 \Rightarrow a_r = 35$$

$$a_1 + a_r + a_w + a_r + a_w = 105 \Rightarrow 5a_w = 105 \Rightarrow a_w = 21$$

$$\Rightarrow a_1 = 105 - 35 - 21 = 49 \Rightarrow a_n = 2 + (n-1) \times 3 \Rightarrow a_{10} = 2 + 9 \times 3 = 29$$

$$S_q = 9S_r \Rightarrow \frac{q}{r}(a_1 + a_q) = 9 \left[\frac{r}{r}(a_1 + a_r) \right] \Rightarrow \frac{a_1 + a_q}{q + a_r} = 9$$

$$2a_1 + 18d = 2a_1 + 36q + 4d = 9a_1 + 4d \Rightarrow 7a_1 = 2d \Rightarrow d = 7a_1$$

$$\Rightarrow a_n = a_1 + (n-1) \times 7a_1 \Rightarrow \frac{a_{10}}{a_1} = \frac{a_1 + (19 \times 7a_1)}{a_1 + (4 \times 7a_1)} = \frac{140a_1}{31a_1} = \frac{140}{31}$$

$$d = \frac{t_n - t_m}{n - m} = \frac{25 - 11}{7 - 1} = \frac{14}{6} = \frac{7}{3} \Rightarrow t_7 = 11 + (3 \times \frac{7}{3}) = 20$$

$$t_1 = 18, t_7 = 20 \Rightarrow d = \frac{20 - 18}{6} = \frac{2}{6} = \frac{1}{3}$$

$$d = \frac{b - a}{n + 1} \Rightarrow -a = \frac{13 - 18}{n + 1} \Rightarrow n + 1 = 10 \Rightarrow n = 9$$