

۱۷,۱۵

الف) $t_n = 2n^2 - 1 \rightarrow$ ① ② ③ ④ ⑤
 $1, 11, 26, 47, 74$ ✓

ب) $t_n = \frac{2n+1}{n+3} \rightarrow$ ① ② ③ ④ ⑤
 $\frac{3}{8}, \frac{5}{6}, \frac{7}{1}, \frac{9}{8}, \frac{11}{9}$ ✓ ②

الف) $t_n = \frac{(-1)^n}{\sqrt{n+3}} \rightarrow t_{13} = \frac{-1}{\sqrt{16}} = -\frac{1}{4}$ ✓ ②

ب) $t_n = 2n - \left[\frac{n}{4}\right] \rightarrow t_{13} = 26 - \left[\frac{13}{4}\right] = 23$ ✓ ②
 $\frac{13}{4} = 3\frac{1}{4} = 3 \times 4$

الف) $2n - 1 < 0 \rightarrow 2n < 1 \rightarrow n < \frac{1}{2} \rightarrow 1 \leq n < 8 \Rightarrow$
 $8 - 1 = 7 \rightarrow$ ۷ جمله منفی ✓

ب) $n^2 - 14n + 40 < 0 \rightarrow (n-4)(n-10) < 0 \Rightarrow \begin{cases} n=4 \\ n=10 \end{cases}$ ②
 $\begin{matrix} 4 & 10 \\ + & - \\ 0 & 0 \end{matrix} \Rightarrow 4 < n < 10 \Rightarrow 5 \leq n \leq 9$
 ۵ جمله منفی: $(9-5)+1 = 5 \rightarrow$ ۵ جمله منفی ✓

الف) $2n - 14 \leq 0 \rightarrow 2n \leq 14 \rightarrow n \leq 7 \rightarrow 1 \leq n \leq 7$
 $(7-1)+1 = 7 \rightarrow$ ۷ جمله مثبت ✓ ②

ب) $n^2 - 14n + 47 \leq 0 \rightarrow (n-3)(n-9) \leq 0 \Rightarrow \begin{cases} n=3 \\ n=9 \end{cases}$ ②
 $\begin{matrix} 3 & 9 \\ + & - \\ 0 & 0 \end{matrix} \Rightarrow 3 \leq n \leq 9$
 $(9-3)+1 = 7 \rightarrow$ ۷ جمله مثبت ✓

$\begin{cases} t_4 = 7 & t_4 = a + 3d & 7 = a + 3d \\ t_9 = 22 & t_9 = a + 8d & 22 = a + 8d \end{cases} \rightarrow \begin{cases} 9 - 4 = 5 & 22 - 7 = 15 \\ 18 \div 5 = 3 = d \end{cases}$

$d = 3 \Rightarrow a = -2$
 $t_4 = a + 3d \Rightarrow -2 + 3(3) = 7$ ✓ ②

$$n^2 - 9n + 2$$

$$\min \rightarrow a > 0 \quad \text{مشتق اول: } n = -\frac{b}{2a} = \frac{9}{2}$$

$$a = \frac{1}{2} \rightarrow \frac{1}{2}n^2 - \frac{9}{2}n + 2 = \boxed{-\frac{5}{2}} \quad \checkmark$$

(1)

$$n^2 + 4n + 1 \quad \min \rightarrow a > 0 \quad \text{مشتق اول: } n = -\frac{b}{2a} = -2 \quad \times \rightarrow \text{از اینجا شروع می‌کنیم}$$

$$\frac{1}{2}n^2 + 4n + 1 = \boxed{-\frac{9}{2}} \quad \min \text{ در } n = -2 \rightarrow \text{جمله اول} \rightarrow t_1 = 9$$

$$t_n = -4, -7, -10, -13, -16, \dots$$

$$b = -4, c = 1 \quad c + b = -3$$

$$\begin{cases} a_1 = -4 = a(1) + b(1) + c = -4 \\ a_2 = -7 = a(2) + b(2) + c = -7 \end{cases} \Rightarrow \begin{cases} -b - c = -8 \\ 2b + c = -11 \end{cases} \Rightarrow \begin{cases} b = -19 \\ c = 21 \end{cases}$$

$$t_n = n^2 - 4n + 1 \quad t_1 = 1$$

$$1(1) - 19(1) + 21(1) = -4 + 21 = \boxed{17}$$

$$t_n = -9, -1, 7, 13, 19, \dots$$

$$a_n = an^2 + bn + c \Rightarrow \begin{cases} a_1 \Rightarrow a + b + c = -9 \\ a_2 \Rightarrow 4a + 2b + c = -1 \\ a_3 \Rightarrow 9a + 3b + c = 1 \end{cases}$$

$$\begin{aligned} (2a + 2b + c) - (a + b + c) &= -1 - (-9) \Rightarrow a + b = 8 \\ (3a + 3b + c) - (2a + 2b + c) &= 1 - (-1) \Rightarrow a + b = 2 \end{aligned}$$

$$2a = 8 \Rightarrow \boxed{a = 4}$$

$$3a + b = 2 \Rightarrow 3(4) + b = 2 \Rightarrow \boxed{b = -10}$$

$$a + b + c = -9 \Rightarrow \boxed{c = 1}$$

$$a_n = 4n^2 - 10n + 1$$

$$(-\infty, a+2], [2a+1, +\infty) \Rightarrow a+2 \geq 2a+1 \Rightarrow 1 \geq a \quad \checkmark$$

$$\boxed{a = 2 \rightarrow \text{فقط نقطه}}$$

$$\boxed{a < 2 \rightarrow \text{هیچ نقطه}}$$

$$a > 2 \quad \checkmark$$

$$\hookrightarrow [a+2, 2a+1] \quad \text{بازه خالی}$$

(2)

$$a_n = 99, d = 3, a_1 = 3$$

$$3, 6, 9, \dots \Rightarrow 3n = t_n \quad n = 33 \Rightarrow t_{33} = 99 \quad \checkmark$$

$$a_n = 3 + (n-1) \cdot 3 \Rightarrow 99 = 3 + (n-1) \cdot 3$$

(1, 8)

$$8, 10, 12, \dots \Rightarrow 2n = t_n \quad n = 19 \Rightarrow t_{19} = 38 \Rightarrow$$

$$a_n = 1, d = 8, a_1 = 1 \Rightarrow a_n = 1 + (n-1) \cdot 8 \Rightarrow 38 = 1 + (n-1) \cdot 8 \Rightarrow n = 5$$

$$18, 20, 22, \dots \Rightarrow 2n = t_n \quad n = 9 \Rightarrow t_9 = 18 \Rightarrow$$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B) = 47$$

$$33 - 9 = 24 \rightarrow \text{اعداد مشترک به غیر}$$

$$19 - 9 = 10 \rightarrow \text{اعداد مشترک به غیر}$$

$$\text{فقط به 8}$$