

الف) $t_n = n^2 - 4n + 2 \Rightarrow t_n = (n-2)^2 - 2 \Rightarrow n-2=0 \Rightarrow n=2$ (2)

$n-2=0 \Rightarrow n=2 \Rightarrow \boxed{t_2 = -2}$

ب) $t_n = (n^2 + 4n + 4) - 4 + 1 \Rightarrow t_n = (n+2)^2 - 3 \Rightarrow (n+2) \geq 0 \Rightarrow n \geq -2$

کمترین مقدار $n+2=0 \Rightarrow n=-2 \rightarrow n=1$ (1)

$-4, -1, 0, 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, 289, 324, 361, 400, 441, 484, 529, 576, 625, 676, 729, 784, 841, 900, 961, 1024, 1089, 1156, 1225, 1296, 1369, 1444, 1521, 1600, 1681, 1764, 1849, 1936, 2025, 2116, 2209, 2304, 2401, 2500, 2601, 2704, 2809, 2916, 3025, 3136, 3249, 3364, 3481, 3600, 3721, 3844, 3969, 4096, 4225, 4356, 4489, 4624, 4761, 4900, 5041, 5184, 5329, 5476, 5625, 5776, 5929, 6084, 6241, 6400, 6561, 6724, 6889, 7056, 7225, 7396, 7569, 7744, 7921, 8100, 8281, 8464, 8649, 8836, 9025, 9216, 9409, 9604, 9801, 10000$

$a_n = n^2 + bn + c \Rightarrow n^2 + bn + c \xrightarrow{z=1} t_1 = 1 + b + c = -4 \Rightarrow b + c = -5$

$t_2 = 4 + b + c = -2 \Rightarrow b + c = -6$

$a_n = n^2 - 4n + 1 \Rightarrow$

مربع کامل

$-b = 11 - a = 6$

$a_{10} = 10^2 - 4 \cdot 10 + 1 = 51$

if $b = -4 \Rightarrow \boxed{c = 1}$

$\Rightarrow \boxed{b = -6}$

$-4, -1, 0, 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, 289, 324, 361, 400, 441, 484, 529, 576, 625, 676, 729, 784, 841, 900, 961, 1024, 1089, 1156, 1225, 1296, 1369, 1444, 1521, 1600, 1681, 1764, 1849, 1936, 2025, 2116, 2209, 2304, 2401, 2500, 2601, 2704, 2809, 2916, 3025, 3136, 3249, 3364, 3481, 3600, 3721, 3844, 3969, 4096, 4225, 4356, 4489, 4624, 4761, 4900, 5041, 5184, 5329, 5476, 5625, 5776, 5929, 6084, 6241, 6400, 6561, 6724, 6889, 7056, 7225, 7396, 7569, 7744, 7921, 8100, 8281, 8464, 8649, 8836, 9025, 9216, 9409, 9604, 9801, 10000$

$a_n = 2n^2 + bn + c \Rightarrow$

$a_1 = 2 + b + c = -4 \Rightarrow b + c = -6$

$a_2 = 8 + b + c = -1 \Rightarrow b + c = -9$

if $b = -1 \Rightarrow \boxed{c = -5}$

$\Rightarrow \boxed{b = -1}$

مجموعه a : $(-\infty, 4)$

$2a+1 > a+2 \Rightarrow 2a-a > 2-1 \Rightarrow \boxed{a > 1} \Rightarrow \text{حدود } a$

استدلال یک عنصری

$a=3 \leftarrow 2a+1 = a+2$

$\textcircled{1} \cap \textcircled{2} \rightarrow a \geq 3$

الف) $n(3) = \left\lfloor \frac{100}{3} \right\rfloor = 33$

ب) $n(5) = \left\lfloor \frac{100}{5} \right\rfloor = 20$

ج) $n(3) \cap n(5) = n(15) = \left\lfloor \frac{100}{15} \right\rfloor = 6$

د) $n(3) \cup n(5) = n(3) + n(5) - n(15) = 33 + 20 - 6 = 47$

$$\begin{aligned} z_1 &= 3 \times (1)^2 - 1 = 2 \\ z_2 &= 3 \times (2)^2 - 1 = 11 \\ z_3 &= 3 \times (3)^2 - 1 = 24 \end{aligned}$$

$$\begin{aligned} z_4 &= 3 \times (4)^2 - 1 = 47 \\ z_5 &= 3 \times (5)^2 - 1 = 74 \end{aligned}$$

$$\begin{aligned} z_1 &= \frac{(2 \times 1) + 1}{2} = \frac{3}{2} \\ z_2 &= \frac{(2 \times 2) + 1}{2} = \frac{5}{2} \\ z_3 &= \frac{(2 \times 3) + 1}{2} = \frac{7}{2} \end{aligned}$$

$$\begin{aligned} z_4 &= \frac{(2 \times 4) + 1}{2} = \frac{9}{2} \\ z_5 &= \frac{(2 \times 5) + 1}{2} = \frac{11}{2} \end{aligned}$$

$$z_{13} = \frac{(-1)^{13}}{\sqrt{13+3}} = \frac{-1}{\sqrt{16}} = \boxed{-\frac{1}{4}}$$

$$z_{13} = 4 \times 13 - \left[\frac{13}{4} \right] = 24 - [3.25] = 24 - 3 = \boxed{21}$$

$$2n - 10 < 0 \Rightarrow 2n < 10 \Rightarrow n < 5 \Rightarrow n = 1, 2, 3, 4 \rightarrow z_1, z_2, z_3, z_4$$

$$\begin{aligned} n^2 - 14n + 40 < 0 &\Rightarrow (n-4)(n-10) < 0 \Rightarrow 4 < n < 10 \\ \Rightarrow n = 5, 6, 7, 8, 9 &\Rightarrow z_5, z_6, z_7, z_8, z_9 \end{aligned}$$

$$\begin{aligned} 2n - 14 < 0 &\Rightarrow 2n < 14 \Rightarrow n < 7 \Rightarrow n = 1, 2, \dots, 6 \\ \Rightarrow z_1, z_2, z_3, \dots, z_6, z_7 \end{aligned}$$

$$\begin{aligned} n^2 - 12n + 27 < 0 &\Rightarrow (n-3)(n-9) < 0 \Rightarrow 3 < n < 9 \\ n = 4, 5, 6, 7, 8 &\Rightarrow z_4, z_5, \dots, z_8, z_9 \end{aligned}$$

$$\begin{aligned} z_4 &= 7 = z_1 + 3d \\ z_9 &= 22 = z_1 + 8d \Rightarrow d = \frac{z_9 - z_4}{9 - 4} = \frac{22 - 7}{5} = \frac{15}{5} = 3 \end{aligned}$$

$$\Rightarrow z_1 + (3 \times 4) = 7 \Rightarrow z_1 = -5$$

$$\Rightarrow z_4 = z_1 + 3d = -5 + (3 \times 3) = 4 \Rightarrow z_4 = 4$$