

الف) $y = x^2 - 5 \rightarrow x^2 = y + 5 \rightarrow x = \pm \sqrt{y+5} \rightarrow y+5 \geq 0 \rightarrow y \geq -5$
 $\rightarrow R_f = [-5, +\infty)$

ب) $y = x^3 + 1 \rightarrow x^3 = y - 1 \rightarrow x = \sqrt[3]{y-1} \rightarrow R_f = \mathbb{R}$

الف) $y = x^2 - 2x + 2 \rightarrow y = x^2 - 2x + 1 + 1 \rightarrow (x-1)^2 + 1 = y \rightarrow (x-1)^2 = y-1$
 $\rightarrow x-1 = \pm \sqrt{y-1} \rightarrow x = \pm \sqrt{y-1} + 1 \rightarrow y-1 \geq 0 \rightarrow y \geq 1 \rightarrow R_f = [1, +\infty)$

ب) $y = x^2 - 5x + 1 \rightarrow y = x^2 - 5x + \frac{25}{4} - \frac{21}{4} \rightarrow y + \frac{21}{4} = (x - \frac{5}{2})^2 \rightarrow x = \pm \sqrt{y + \frac{21}{4}} + \frac{5}{2}$
 $\rightarrow y + \frac{21}{4} \geq 0 \rightarrow y \geq -\frac{21}{4} \rightarrow R_f = [-\frac{21}{4}, +\infty)$

الف) $y = \frac{x^2+3}{x^2-1} \rightarrow x^2 = \frac{-(-2y-3)}{y-1} = \frac{2y+3}{y-1} \rightarrow x = \pm \sqrt{\frac{2y+3}{y-1}}$
 $\rightarrow \frac{2y+3}{y-1} \geq 0 \rightarrow \frac{-\frac{3}{2}}{\frac{1}{2}} \rightarrow R_f = (-\infty, -\frac{3}{2}] \cup (1, +\infty)$

ب) $y = \frac{1|x|+1}{|x|-1} \rightarrow |x| = \frac{-(-2y-1)}{y-1} = \frac{2y+1}{y-1} \rightarrow \frac{2y+1}{y-1} \geq 0 \rightarrow \frac{-\frac{1}{2}}{\frac{1}{2}} \rightarrow R_f = (-\infty, -\frac{1}{2}] \cup (1, +\infty)$

$y = \frac{1}{x^2-2x} \rightarrow yx^2 - 2yx - 1 = 0 \rightarrow x = \frac{2y \pm \sqrt{4y^2+4y}}{2y} \rightarrow 4y^2+4y \geq 0$

$\rightarrow 4y(y+1) \geq 0 \rightarrow \frac{-1}{4} \rightarrow R_f = (-\infty, -\frac{1}{4}] \cup (0, +\infty)$

الف) $y = x^2 - 4x + 2 \rightarrow \min \text{ at } \left| \frac{-b}{2a} = 2 \rightarrow y = -2 \rightarrow R_f = [-2, +\infty) \right.$

ب) $y = -x^2 + 2x + 2 \rightarrow \max \text{ at } \left| \frac{-b}{2a} = 1 \rightarrow y = 3 \rightarrow R_f = (-\infty, 3] \right.$

ا) $y = \sqrt{x^2 - 4x + 4} \rightarrow \min \text{ ext } \left| \begin{array}{l} \frac{-b}{2a} = 1 \\ y = -1 \end{array} \right. \rightarrow [-1, +\infty) \xrightarrow{\text{انزياح}} [0, +\infty)$

ب) $y = \sqrt{-x^2 + 4x + 1} \rightarrow \max \text{ ext } \left| \begin{array}{l} \frac{-b}{2a} = 2 \\ y = 1 \end{array} \right. \rightarrow (-\infty, 1] \xrightarrow{\text{انزياح}} R_f = [0, \sqrt{17}]$

ا) $y = x^4 + 3x^2 + 2x + 1 \rightarrow R_f = \mathbb{R}$

ب) $y = \sqrt{x^4 + 2x^2 + 4x + 1} \rightarrow R_f = [0, +\infty)$

ا) $y = \frac{3x+1}{x-2} \rightarrow R_f = \mathbb{R} - \{3\}$

ب) $y = \sqrt{\frac{3x+1}{x+3}} \rightarrow \mathbb{R} - \{3\} \xrightarrow{\text{انزياح}} R_f = [0, +\infty) - \{3\}$

ا) $y = \frac{3x+1}{x-2} \rightarrow \begin{array}{l} \text{مختبئ} = 2 \\ \text{مختبئ} = 3 \end{array} \xrightarrow{y=0} y = -\frac{1}{3}$

ب) $y = \frac{2x-2}{1-2x} = \frac{-2(1-x)}{1-2x} = -2$

$x \neq \frac{1}{2}$

ا) $y = \cos^2 x + \frac{1}{\cos^2 x} \xrightarrow{\cos^2 x \in [0, 1]} [2, +\infty)$

ب) $y = \sqrt{\frac{2x+1}{x}} \xrightarrow{x > 0, x \neq -\frac{1}{2}} (-\infty, -2] \cup [2, +\infty) \xrightarrow{\text{انزياح}} (-\infty, \sqrt{-2}] \cup [\sqrt{2}, +\infty)$