

الف) $y = x^2 - 5 \rightarrow x^2 = y + 5 \rightarrow x = \pm \sqrt{y + 5} \rightarrow y + 5 \geq 0 \rightarrow y \geq -5$
 $\rightarrow R_f = [-5, +\infty)$ ✓

ب) $y = x^3 + 1 \rightarrow x^3 = y - 1 \rightarrow x = \sqrt[3]{y - 1} \rightarrow R_f = \mathbb{R}$ ✓

الف) $y = x^2 - 2x + 2 \rightarrow y = x^2 - 2x + 1 + 1 \rightarrow (x - 1)^2 + 1 = y \rightarrow (x - 1)^2 = y - 1$
 $\rightarrow x - 1 = \pm \sqrt{y - 1} \rightarrow x = \pm \sqrt{y - 1} + 1 \rightarrow y - 1 \geq 0 \rightarrow y \geq 1 \rightarrow R_f = [1, +\infty)$ ✓

ب) $y = x^2 - 5x + 1 \rightarrow y = x^2 - 5x + \frac{25}{4} - \frac{25}{4} + 1 \rightarrow y + \frac{21}{4} = (x - \frac{5}{2})^2$
 $\rightarrow x - \frac{5}{2} = \pm \sqrt{y + \frac{21}{4}} \rightarrow x = \pm \sqrt{y + \frac{21}{4}} + \frac{5}{2}$
 $\rightarrow y + \frac{21}{4} \geq 0 \rightarrow y \geq -\frac{21}{4} \rightarrow R_f = [-\frac{21}{4}, +\infty)$ ✓

الف) $y = \frac{x^2 + 3}{x^2 - 1} \rightarrow x^2 = \frac{-(-2y - 3)}{y - 1} = \frac{2y + 3}{y - 1} \rightarrow x = \pm \sqrt{\frac{2y + 3}{y - 1}}$
 $\rightarrow \frac{2y + 3}{y - 1} \geq 0 \rightarrow \frac{-\frac{3}{2}}{1 - \frac{1}{y}} \rightarrow R_f = (-\infty, -\frac{3}{2}] \cup (1, +\infty)$ ✓

ب) $y = \frac{1}{|x| - 1} \rightarrow |x| = \frac{-(-2y - 1)}{y - 2} = \frac{2y + 1}{y - 2} \rightarrow \frac{2y + 1}{y - 2} \geq 0 \rightarrow \frac{-\frac{1}{2}}{1 - \frac{2}{y}} \rightarrow R_f = (-\infty, -\frac{1}{2}] \cup (2, +\infty)$ ✓

$y = \frac{1}{x^2 - 2x} \rightarrow yx^2 - 2yx - 1 = 0 \rightarrow x = \frac{2y \pm \sqrt{4y^2 + 4y}}{2y} \rightarrow 4y^2 + 4y \geq 0$
 $\rightarrow 4y(y + 1) \geq 0 \rightarrow \frac{-\frac{1}{4}}{1 - \frac{2}{y}} \rightarrow R_f = (-\infty, -\frac{1}{4}] \cup (0, +\infty)$ ✓

الف) $y = x^2 - 4x + 2 \rightarrow \min \text{ ext } \left| \frac{b}{2a} = 2 \rightarrow y = -2 \rightarrow R_f = [-2, +\infty) \right.$ ✓

ب) $y = -x^2 + 2x + 2 \rightarrow \max \text{ ext } \left| \frac{b}{2a} = 1 \rightarrow y = 4 \rightarrow R_f = (-\infty, 4] \right.$ ✓

ا) $y = \sqrt{x^2 - 4x + 4} \rightarrow \min \text{ ext} \left| \begin{array}{l} \frac{-b}{2a} = 1 \\ y = -1 \end{array} \right. \rightarrow [-1, +\infty) \xrightarrow{\text{انزياح}} [0, +\infty)$ $R_2 = [0, +\infty)$ ✓

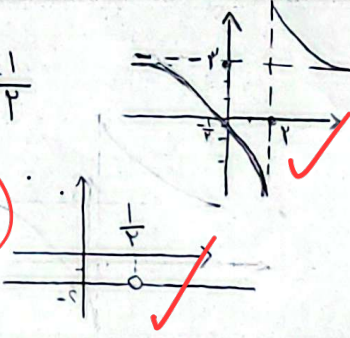
ب) $y = \sqrt{-x^2 + 2x + 1} \rightarrow \max \text{ ext} \left| \begin{array}{l} \frac{-b}{2a} = 1 \\ y = 1 \end{array} \right. \rightarrow (-\infty, 1] \xrightarrow{\text{انزياح}} R_2 = [0, \sqrt{2}]$ ✓ (2)

ا) $y = x^4 + 3x^2 + 2x + 1 \rightarrow R_2 = \mathbb{R}$ ✓

ب) $y = \sqrt{x^4 + 2x^2 + 4x + 1} \rightarrow R_2 = [0, +\infty)$ ✓ (2)

ا) $y = \frac{3x+1}{x-2} \rightarrow R_2 = \mathbb{R} - \{2\}$ ✓

ب) $y = \sqrt{\frac{3x+1}{x+3}} \rightarrow \mathbb{R} - \{2\} \xrightarrow{\text{انزياح}} R_2 = [0, +\infty) - \{2\}$ ✓ (2)

ا) $y = \frac{3x+1}{x-2} \rightarrow \begin{array}{l} \text{مختار} = (2) \rightarrow \text{مختار} = (3) \rightarrow \frac{y}{x} \end{array} \xrightarrow{u=0} y = -\frac{1}{2}$
 ب) $y = \frac{3x-2}{1-2x} = \frac{-2(1-\frac{1}{2}x)}{1-2x} = -2$
 $x \neq \frac{1}{2}$
 ✓ (2)

ا) $y = \cos^2 x + \frac{1}{\cos^2 x} \xrightarrow{\cos^2 x \in [0, 1]} [2, +\infty)$ ✓ (2)

ب) $y = \sqrt{\frac{2x+1}{x}} \xrightarrow{x > 0, x \leq 1} (-\infty, -2] \cup [2, +\infty) \xrightarrow{\text{انزياح}} (-\infty, \sqrt{-2}] \cup [\sqrt{2}, +\infty)$ ✓ (2)