

الف) $y = x^2 - 5 \Rightarrow x^2 = y + 5 \Rightarrow x = \pm \sqrt{y+5} \Rightarrow y \geq -5$
 $y+5 \geq 0 \Rightarrow R_f = [-5, +\infty)$

ب) $y = x^2 + 1 \Rightarrow x^2 = y - 1 \Rightarrow x = \pm \sqrt{y-1}$
 $R_f = \mathbb{R}$
 محدودیتی ندارد

الف) $x^2 - 4x + 4 = y \Rightarrow (x-2)^2 = y-4 \Rightarrow x-2 = \pm \sqrt{y-4} \Rightarrow x = \pm \sqrt{y-4} + 2$
 $(x^2 - 4x + 4)^2 = y \Rightarrow x-2 = \pm \sqrt{y-4} \Rightarrow y-4 \geq 0 \Rightarrow y \geq 4 \Rightarrow R_f = [4, +\infty)$

ب) $y = x^2 - 5x + 1 \Rightarrow (x - \frac{5}{2})^2 = y + \frac{25}{4} - 1 \Rightarrow y + \frac{25}{4} - 1 \geq 0 \Rightarrow y \geq -\frac{21}{4}$
 $\frac{25}{4} - 1 + y = x^2 - 5x + 1 + \frac{25}{4} - 1 \Rightarrow x = \pm \sqrt{y + \frac{21}{4}} + \frac{5}{2}$
 $R_f = [-\frac{21}{4}, +\infty)$

الف) $y = \frac{x+3}{x^2-2} \Rightarrow x^2y - 2y = x+3 \Rightarrow x^2y - x = 2y+3 \Rightarrow x(y-1) = 2y+3$
 $\Rightarrow x = \frac{2y+3}{y-1} \Rightarrow x = \pm \sqrt{\frac{2y+3}{y-1}} \Rightarrow \left\{ \frac{-\frac{3}{2}}{1+\frac{1}{2}} - \frac{1}{\frac{1}{2}} \right\} R_f = (-\infty, -\frac{3}{2}] \cup (1, +\infty)$

ب) $y = \frac{x|x|+1}{|x|-4} \Rightarrow |x|y - y = x|x|+1 \Rightarrow |x|(y-2|x|) = y+1 \Rightarrow |x|(y-2|x|) = y+1$
 $\Rightarrow |x| = \frac{y+1}{y-2} \Rightarrow \left\{ \frac{-\frac{1}{2}}{1+\frac{1}{2}} - \frac{2}{\frac{1}{2}} \right\} R_f = (-\infty, -\frac{1}{2}] \cup (2, +\infty)$

$y = \frac{1}{x^2-4x} \Rightarrow x^2 - 4x + 4 = \frac{1}{y} + 4 \Rightarrow (x-2)^2 = \frac{1}{y} + 4$
 $\Rightarrow x = \pm \sqrt{\frac{1}{y} + 4} + 2 \Rightarrow \frac{1}{y} + 4 \geq 0 \Rightarrow \frac{1+\frac{4}{y}}{y} \geq 0 \Rightarrow \left\{ \frac{-\frac{1}{4}}{1+\frac{1}{4}} - \frac{4}{\frac{1}{4}} \right\}$
 $R_f = (-\infty, -\frac{1}{4}] \cup (0, +\infty)$

الف) $y = x^2 - 9x + 2 \Rightarrow \min \left\{ \frac{y}{x^2} = 3 \right\} \Rightarrow R_f = [-9, +\infty)$

ب) $y = -x^2 + 9x + 2 \Rightarrow \max \left\{ \frac{y}{x^2} = 2 \right\} \Rightarrow R_f = (-\infty, 9]$

$$\text{الف) } y = \sqrt{x^2 - 4x + 4} \left\{ \min \right\}_{-V}^3 \Rightarrow R_f = \sqrt{[-V, +\infty)} = [0, +\infty)$$

$$\text{ب) } y = \sqrt{-x^2 + 4x + 1} \left\{ \max \right\}_{14}^2 \Rightarrow R_f = \sqrt{[-\infty, 14]} = [0, \sqrt{14}]$$

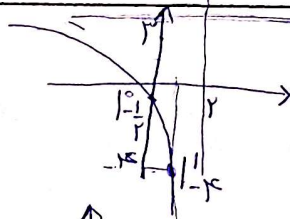
$$\text{الف) } y = \sqrt{x^2 + 3x + 2} \xrightarrow{\text{منزلتین توان فرست}} \Rightarrow R_f = \mathbb{R}$$

$$\text{ب) } y = \sqrt{x^2 + 3x + 2} \Rightarrow R_f = \sqrt{\mathbb{R}} = [0, +\infty)$$

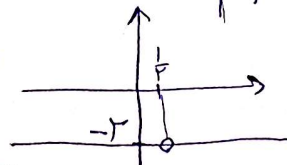
$$\text{الف) } y = \frac{3x+1}{x-2} \Rightarrow R_f = \mathbb{R} - \left\{ \frac{3}{1} \right\} = \mathbb{R} - \{3\}$$

$$\text{ب) } y = \sqrt{\frac{3x+1}{x+3}} \Rightarrow R_f = \sqrt{\mathbb{R} - \{3\}} = [0, +\infty) - \{2\}$$

$$\text{الف) } y = \frac{3x+1}{x-2} \quad \begin{array}{l} x \neq 2 \text{ مخرج مساوی} \\ y \neq 3 \text{ صیغیه ناقص} \end{array}$$



$$\text{ب) } y = \frac{3x-2}{-2x+1} = \frac{-2(-x+1)}{-2x+1} = -2 \quad \begin{array}{l} x \neq \frac{1}{2} \\ y \neq -2 \end{array}$$



$$\text{الف) } y = \cos^2 x + \frac{1}{\cos^2 x} \quad \cos^2 x > 0 \left\{ R_f = [2, +\infty) \right.$$

$$\text{ب) } y = \sqrt{\frac{x^2+1}{x}} \quad \frac{x^2+1}{x} = \frac{x^2}{x} + \frac{1}{x} = x + \frac{1}{x} \rightarrow \begin{cases} \text{if } x > 0 \rightarrow R_f = [2, +\infty) \\ \text{if } x < 0 \rightarrow R_f = (-\infty, -2] \end{cases}$$

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$$\Rightarrow R_f = (-\infty, -2] \cup [2, +\infty)$$

$$R_f = (-\infty, \sqrt{-2}] \cup [\sqrt{2}, +\infty)$$