

الف) $y = x^2 - 5 \rightarrow x^2 = y + 5$

$x = \sqrt{y+5} \quad y+5 \geq 0 \quad y \geq -5$

$R_f = [-5, +\infty)$

ب) $y = x^2 + 1$

$x^2 = y - 1$

$x = \sqrt{y-1}$

$R_f = \mathbb{R}$

الف) $y = x^2 - 4x + 7$

$y = x^2 - 4x + 4 + 3 \rightarrow y = (x-2)^2 + 3$

$(x-2)^2 = y - 3$

$x-2 = \sqrt{y-3}$

$x = \sqrt{y-3} + 2$

$y-3 \geq 0$
 $y \geq 3$

$R_f = [3, +\infty)$

ب) $y = x^2 - 5x + 1$

$x^2 - 5x + 1 - y = 0$

$\Delta x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\frac{5 \pm \sqrt{25 - 4(1-y)}}{2} \Rightarrow$

$25 - 4(1-y) \geq 0$

$\frac{25}{4} - 1 + 4y \geq 0$

$4y \geq -21$

$y \geq -\frac{21}{4}$

$R_f = [-\frac{21}{4}, +\infty)$

الف) $y = \frac{x^2 + 3}{x^2 - 2}$

$x^2 + 3 = yx^2 - 2y$

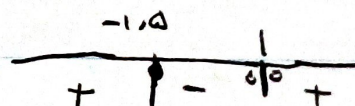
$x^2 - yx^2 = -2y - 3$

$x^2(1-y) = -2y-3$

$x^2 = \frac{-2y-3}{1-y}$

$x = \sqrt{\frac{-2y-3}{1-y}}$

$\begin{cases} y = \frac{-5}{2} \\ y = 1 \end{cases}$



$R_f = (-\infty, -1.5] \cup (1, +\infty)$

$$\Rightarrow y = \frac{r|x|+1}{|x|-\varepsilon}$$

$$r|x|+1 = y|x| - \varepsilon y$$

$$r|x| - y|x| = -\varepsilon y - 1$$

$$|x|(r-y) = -(\varepsilon y + 1)$$

$$|x| = \frac{-(\varepsilon y + 1)}{r-y}$$

$$\frac{-(\varepsilon y - 1)}{r-y} \gg \cdot \quad \begin{cases} y = -\frac{1}{\varepsilon} \\ y = r \end{cases}$$

$$K_f = R - (-\frac{1}{\varepsilon}, r]$$

$$\frac{-\frac{1}{\varepsilon}}{r - (-\frac{1}{\varepsilon})}$$

$$y = \frac{1}{x^r - \varepsilon} \rightarrow$$

$$1 = y x^r - \varepsilon y x$$

$$y x^r - \varepsilon y x - 1 = 0$$

$$\Delta \gg \cdot \quad -b \pm \sqrt{b^2 - 4ac} \rightarrow 1 \pm y^r - \varepsilon(-y) \gg \cdot$$

$$\frac{-\frac{1}{\varepsilon}}{r - (-\frac{1}{\varepsilon})}$$

$$1 \pm y^r + \varepsilon y \gg \cdot \quad \begin{cases} y = 0 \\ y = -\frac{1}{\varepsilon} \end{cases}$$

$$K_f = R - (-\frac{1}{\varepsilon}, 0]$$

$$\Rightarrow y = x^r - y x + r \xrightarrow{a > 0} \begin{cases} x_{min} = -\frac{b}{r a} = r \\ y_{min} = -r \end{cases}$$

$$K_f = [-r, +\infty)$$

$$\Rightarrow y = -x^r + \varepsilon x + r \xrightarrow{a < 0} \begin{cases} x_{max} = -\frac{b}{r a} = -\frac{-\varepsilon}{-r} = r \\ y_{max} = r \end{cases}$$

$$K_f = (-\infty, r]$$

$$\Rightarrow y = \sqrt{x^r - y x + r} \xrightarrow{a > 0} \begin{cases} x_{min} = -\frac{b}{r a} = r \\ y_{min} = -r \end{cases}$$

$$K_f = [-r, +\infty) \rightarrow K_f = [0, +\infty)$$

$$\Rightarrow y = \sqrt{-x^r + \varepsilon x + 1} \xrightarrow{a < 0} \begin{cases} x_{max} = -\frac{b}{r a} = r \\ y_{max} = 1 \end{cases}$$

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$$K_f = \sqrt{(-\infty, 1 \varepsilon]} \rightarrow K_f = [0, \sqrt{1 \varepsilon}]$$

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$$الف) y = x^4 + 3x^2 + 2x + 1$$

$$R_f = \mathbb{R}$$

-1

$$ب) R_f = \sqrt{\mathbb{R}} \Rightarrow R_f = [0, +\infty)$$

$$الف) y = \frac{3x+1}{x-2}$$

$$R_f = \mathbb{R} - \{2\}$$

-1

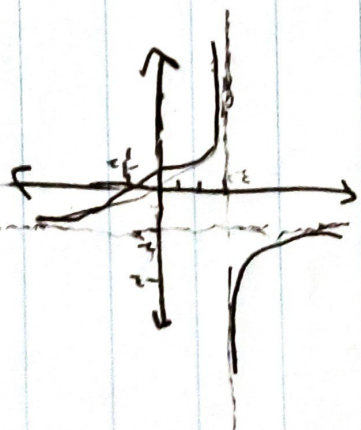
$$ب) y = \sqrt{\frac{x^2+1}{x+3}}$$

$$R_f = \sqrt{\mathbb{R} - \{-3\}}$$

$$\rightarrow R_f = [0, +\infty) - \{1\}$$

$$الف) y = \frac{3x+1}{x-2}$$

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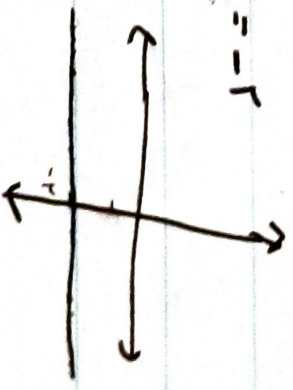


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$$ب) y = \frac{x^2-1}{1-2x}$$

$$\rightarrow y = \frac{-1(-2x+1)}{1-2x}$$

$$\rightarrow y = -1$$



$$الف) y = \cos^{-1} x + \frac{1}{\cos^{-1} x}$$

$$R_f = [1, +\infty)$$

-10

$$ب) y = \sqrt{\frac{x^2+1}{x}}$$

$$\rightarrow y = \sqrt{x + \frac{1}{x}}$$

$$R_f = (-\infty, -1] \cup [1, +\infty)$$