

الف)  $y = x^2 - 5 \rightarrow x^2 = y + 5$

$x = \sqrt{y+5} \quad y+5 \geq 0 \quad y \geq -5$

$R_f = [-5, +\infty)$

ب)  $y = x^2 + 1$

$x^2 = y - 1$

$x = \sqrt{y-1}$

$R_f = \mathbb{R}$

الف)  $y = x^2 - 4x + 7$

$y = x^2 - 4x + 4 + 3 \rightarrow y = (x-2)^2 + 3$

$(x-2)^2 = y - 3$

$x-2 = \sqrt{y-3}$

$x = \sqrt{y-3} + 2$

$y-3 \geq 0$   
 $y \geq 3$

$R_f = [3, +\infty)$

ب)  $y = x^2 - 5x + 1$

$x^2 - 5x + 1 - y = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\frac{5 \pm \sqrt{25 - 4(1-y)}}{2} \Rightarrow$

$25 - 4(1-y) \geq 0$

$\frac{25}{4} - 1 + 4y \geq 0$

$4y \geq -21$

$y \geq -\frac{21}{4}$

$R_f = [-\frac{21}{4}, +\infty)$

الف)  $y = \frac{x^2 + 3}{x^2 - 2}$

$x^2 + 3 = yx^2 - 2y$

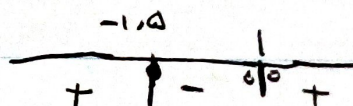
$x^2 - yx^2 = -2y - 3$

$x^2(1-y) = -2y-3$

$x^2 = \frac{-2y-3}{1-y}$

$x = \sqrt{\frac{-2y-3}{1-y}}$

$\begin{cases} y = \frac{-5}{2} \\ y = 1 \end{cases}$



$R_f = (-\infty, -1.5] \cup (1, +\infty)$



$$\Rightarrow y = \frac{r|x|+1}{|x|-\varepsilon}$$

$$r|x|+1 = y|x| - \varepsilon y$$

$$r|x| - y|x| = -\varepsilon y - 1$$

$$|x|(r-y) = -(\varepsilon y + 1)$$

$$|x| = \frac{-(\varepsilon y + 1)}{r-y}$$

$$\frac{-(\varepsilon y + 1)}{r-y} > 0 \quad \begin{cases} y = -\frac{1}{\varepsilon} \\ y = r \end{cases}$$

$$K_f = rR - (-\frac{1}{\varepsilon} \circ r) \quad \checkmark \quad + \frac{-\frac{1}{\varepsilon}}{r} - \frac{r}{0^0+}$$

$$y = \frac{1}{x^r - \varepsilon x} \rightarrow$$

$$1 = y x^r - \varepsilon y x$$

$$y x^r - \varepsilon y x - 1 = 0$$

$$\Delta > 0 \quad -b^r - \varepsilon a c \rightarrow 1 + y^r - \varepsilon(-y) > 0$$

$$\frac{-1 \pm \sqrt{1 - 4(-\varepsilon a c)}}{2(-\varepsilon a)}$$

$$K_f = rR - (-\frac{1}{\varepsilon} \circ r) \quad \checkmark$$

$$1 + y^r + \varepsilon y > 0 \quad \begin{cases} y = 0 \\ y = -\frac{1}{\varepsilon} \end{cases}$$

$$\textcircled{r}$$

$$\Rightarrow y = x^r - y x + r \xrightarrow{a > 0}$$

$$\begin{cases} x_{min} = -\frac{b}{ra} = r \\ y_{min} = -r \end{cases}$$

$$K_f = [-r, +\infty) \quad \checkmark$$

$$\circ \Rightarrow y = -x^r + \varepsilon x + r \xrightarrow{a < 0} \begin{cases} x_{max} = -\frac{b}{ra} = -\frac{\varepsilon}{-r} = r \\ y_{max} = r \end{cases}$$

$$\textcircled{r}$$

$$K_f = (-\infty, r] \quad \checkmark$$

$$\Rightarrow y = \sqrt{x^r - y x + r} \xrightarrow{a > 0}$$

$$\begin{cases} x_{min} = -\frac{b}{ra} = r \\ y_{min} = -r \end{cases}$$

$$K_f = [-r, +\infty) \rightarrow K_f = [0, +\infty) \quad \checkmark$$

$$\Rightarrow y = \sqrt{-x^r + \varepsilon x + 1} \xrightarrow{a < 0}$$

$$\begin{cases} x_{max} = -\frac{b}{ra} = r \\ y_{max} = r \end{cases}$$

$$\textcircled{r}$$

$$K_f = \sqrt{(-\infty, 1]} \rightarrow K_f = [0, \sqrt{1}] \quad \checkmark$$

GOLBARG



Subject:  
Year.

Month.

Date.

|    |    |    |    |    |    |    |
|----|----|----|----|----|----|----|
| Sa | Su | Mo | Tu | We | Th | Fr |
|----|----|----|----|----|----|----|

الف)  $y = x^2 + 3x^2 + 1$

$R_f = \mathbb{R}$  ✓

ب)  $R_f = \sqrt{\mathbb{R}} \Rightarrow R_f = [0, +\infty)$  ✓

الف)  $y = \frac{3x+1}{x-1}$

$R_f = \mathbb{R} - \{1\}$  ✓

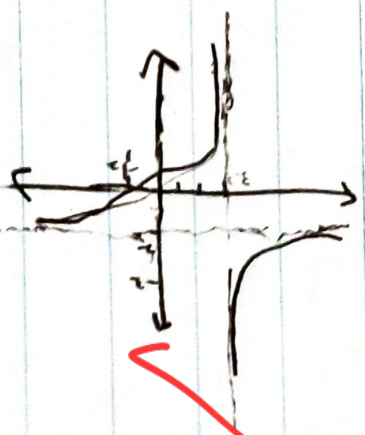
ب)  $y = \sqrt{\frac{x^2+1}{x+3}}$

$R_f = \sqrt{\mathbb{R} - \{1\}}$

$\rightarrow R_f = [-1, +\infty) - \{1\}$  ✓

الف)  $y = \frac{3x+1}{x-1}$

منايات  $x = 1$   
منايات  $y = 4$



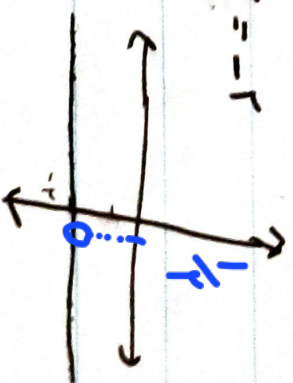
(1, 4) ✓

ب)  $y = \frac{x^2-1}{1-x}$

$\rightarrow y = \frac{-1(-1-x+1)}{1-x}$

$\rightarrow y = -1$

$x \neq 1$



الف)  $y = \cos^{-1} x + \frac{1}{\cos^{-1} x}$

$R_f = [1, +\infty)$  ✓

(1, 1) ✓

ب)  $y = \sqrt{\frac{x^2+1}{x}}$

$\rightarrow y = \sqrt{x + \frac{1}{x}}$

$R_f = [-\infty, -1] \cup [1, +\infty)$

$R_f = (-\infty, \sqrt{-1}] \cup [\sqrt{1}, +\infty)$