

الف) $y = x^2 - 5 \Rightarrow x^2 = y + 5 \Rightarrow x = \pm \sqrt{y+5} \Rightarrow R_f = [-5, +\infty)$

ب) $y = x^2 + 1 \Rightarrow x^2 = y - 1 \Rightarrow x = \pm \sqrt{y-1} \Rightarrow R_f = [1, +\infty)$

الف) $y = x^2 - 4x + 7 \Rightarrow x^2 - 4x + 4 + 3 = y \Rightarrow (x-2)^2 + 3 = y \Rightarrow (x-2)^2 = y-3 \Rightarrow x-2 = \pm \sqrt{y-3} \Rightarrow x = 2 \pm \sqrt{y-3} \Rightarrow R_f = [3, +\infty)$

ب) $y = x^2 - 5x + 1 \Rightarrow y + \frac{25}{4} = x^2 - 5x + 1 + \frac{25}{4} = (x - \frac{5}{2})^2 \Rightarrow x - \frac{5}{2} = \pm \sqrt{y + \frac{21}{4}} \Rightarrow x = \frac{5}{2} \pm \sqrt{y + \frac{21}{4}} \Rightarrow R_f = [-\frac{21}{4}, +\infty)$

الف) $y = \frac{x^2 + 2}{x^2 - 2} \Rightarrow y(x^2 - 2) = x^2 + 2 \Rightarrow yx^2 - 2y = x^2 + 2 \Rightarrow yx^2 - x^2 = 2y + 2 \Rightarrow x^2(y-1) = 2y+2 \Rightarrow x^2 = \frac{2y+2}{y-1} \Rightarrow x = \pm \sqrt{\frac{2y+2}{y-1}} \Rightarrow R_f = (-\infty, \frac{1}{2}) \cup (1, +\infty)$

ب) $y = \frac{x|x|+1}{|x|-2} \Rightarrow y(|x|-2) = x|x|+1 \Rightarrow y|x|-2y = x|x|+1 \Rightarrow y|x|-x|x| = 4y+1 \Rightarrow |x|(y-2) = 4y+1 \Rightarrow |x| = \frac{4y+1}{y-2} \Rightarrow x = \pm \frac{4y+1}{y-2} \Rightarrow R_f = (-\infty, \frac{1}{2}) \cup (1, +\infty)$

الف) $y = \frac{1}{x^2 - 4x} \Rightarrow y(x^2 - 4x) = 1 \Rightarrow yx^2 - 4yx - 1 = 0 \Rightarrow x = \frac{4y \pm \sqrt{16y^2 + 4y}}{2y} = \frac{2y \pm \sqrt{4y^2 + y}}{y} \Rightarrow 4y^2 + y \geq 0 \Rightarrow y(4y+1) \geq 0 \Rightarrow y \leq -\frac{1}{4} \text{ or } y \geq 0 \Rightarrow R_f = (-\infty, -\frac{1}{4}] \cup (0, +\infty)$

ب) $y = -x^2 - 4x + 2 \Rightarrow -x^2 - 4x + 2 = y \Rightarrow x^2 + 4x + y - 2 = 0 \Rightarrow x = \frac{-4 \pm \sqrt{16 - 4(y-2)}}{2} = -2 \pm \sqrt{4 - y + 2} = -2 \pm \sqrt{6-y} \Rightarrow 6-y \geq 0 \Rightarrow y \leq 6 \Rightarrow R_f = (-\infty, 6]$

الف) $y = x^2 - 7x + 2 \Rightarrow x^2 - 7x + 2 = y \Rightarrow x^2 - 7x + (2-y) = 0 \Rightarrow \Delta = 49 - 4(2-y) = 49 - 8 + 4y = 41 + 4y \geq 0 \Rightarrow y \geq -\frac{41}{4} \Rightarrow R_f = [-\frac{41}{4}, +\infty)$

ب) $y = -x^2 - 4x + 2 \Rightarrow -x^2 - 4x + 2 = y \Rightarrow x^2 + 4x + y - 2 = 0 \Rightarrow \Delta = 16 - 4(y-2) = 16 - 4y + 8 = 24 - 4y \geq 0 \Rightarrow y \leq 6 \Rightarrow R_f = (-\infty, 6]$

1) $y = \sqrt{x^2 - 5x + 7} \Rightarrow \begin{cases} \text{max} = \frac{-b}{2a} = \frac{5}{2} \\ \text{min} = \frac{4ac - b^2}{4a} = \frac{4 \cdot 1 \cdot 7 - 25}{4} = -\frac{9}{4} \end{cases}$
 $\Rightarrow R_f = \sqrt{[-\frac{9}{4}, +\infty)}$
 $R_f = [0, +\infty)$

2) $y = \sqrt{-x^2 + 4x + 1} \Rightarrow \begin{cases} \text{max} = \frac{-b}{2a} = \frac{2}{-1} = -2 \\ \text{min} = \frac{4ac - b^2}{4a} = \frac{4 \cdot (-1) \cdot 1 - 16}{-4} = 5 \end{cases}$
 $\Rightarrow R_f = \sqrt{[-\infty, 5]} = [-\infty, \sqrt{5}]$
 $R_f = [-\sqrt{5}, \sqrt{5}]$

3) $y = x^2 + 2x + 1 \Rightarrow \text{parabola opening up}$
 $R_f = [1, +\infty)$

4) $y = \sqrt{x^2 + 4x + 1} \Rightarrow \text{parabola opening up}$
 $R_f = [1, +\infty)$

5) $y = \frac{x+1}{x-2} \Rightarrow R_f = \mathbb{R} - \left\{ \frac{1}{2} \right\}$
 $R_f = \mathbb{R} - \left\{ \frac{1}{2} \right\}$

6) $y = \sqrt{\frac{x+1}{x-2}} \Rightarrow R_f = [0, +\infty) - \left\{ \frac{1}{2} \right\}$
 $R_f = [0, +\infty) - \left\{ \frac{1}{2} \right\}$

7) $y = \frac{x+1}{x-2} \Rightarrow \text{asymptotes: } x=2, y=-1$
 $R_f = \mathbb{R} - \left\{ \frac{1}{2} \right\}$

8) $y = \frac{x-1}{1-2x} \Rightarrow \frac{1}{y} = \frac{x-1}{1-2x}$
 $y = -\frac{1}{2} \Rightarrow x = \frac{1}{2}$
 $R_f = \mathbb{R} - \left\{ \frac{1}{2} \right\}$

9) $y = \cos x + \frac{1}{\cos x} \Rightarrow R_f = [2, +\infty)$
 $R_f = [2, +\infty)$

10) $y = \sqrt{\frac{x+1}{x}} \Rightarrow y = \sqrt{x + \frac{1}{x}}$
 $\Rightarrow R_f = \sqrt{[-\infty, -1]} \cup [1, +\infty)$
 $R_f = (-\infty, -\sqrt{2}] \cup [\sqrt{2}, +\infty)$