

$$y = x^r - \omega \Rightarrow x^r = y + \omega \Rightarrow x = \sqrt[r]{y + \omega} \Rightarrow y + \omega \geq 0 \Rightarrow y \geq -\omega \Rightarrow R_f = [-\omega, +\infty)$$

$$ب) y = x^r + 1 \Rightarrow x^r = y - 1 \Rightarrow x = \sqrt[r]{y - 1} \Rightarrow R_f = [1, +\infty)$$

$$الف) y = x^r - \omega x + \epsilon \Rightarrow y = (x - r)^r + r \Rightarrow (x - r)^r = y - r \Rightarrow x - r = \sqrt[r]{y - r} \Rightarrow x = \sqrt[r]{y - r} + r, y - r \geq 0 \Rightarrow y \geq r \Rightarrow R_f = [r, +\infty)$$

$$ب) y = x^r - \omega x + 1 \Rightarrow y = x^r - \omega x + 1 + \omega/r\omega - \omega/r\omega \Rightarrow y = (x - r/\omega)^r - \omega/r\omega \Rightarrow (x - r/\omega)^r = y + \omega/r\omega \Rightarrow x - r/\omega = \sqrt[r]{y + \omega/r\omega} \Rightarrow x = \sqrt[r]{y + \omega/r\omega} + r/\omega \Rightarrow y + \omega/r\omega \geq 0 \Rightarrow y \geq -\omega/r\omega \Rightarrow R_f = [-\omega/r\omega, +\infty)$$

$$الف) y = \frac{x^r + r}{x^r - r} \Rightarrow y x^r - r y = x^r + r \Rightarrow y x^r - x^r = r + r y \Rightarrow x^r (y - 1) = r + r y \Rightarrow x^r = \frac{r + r y}{y - 1} \Rightarrow x = \sqrt[r]{\frac{r + r y}{y - 1}} \Rightarrow R_f = (-\infty, -\frac{r}{r-1}] \cup (1, +\infty)$$

$$ب) y = \frac{r|x| + 1}{|x| - r} \Rightarrow y|x| - r y = r|x| + 1 \Rightarrow y|x| - r|x| = 1 + r y \Rightarrow |x|(y - r) = 1 + r y \Rightarrow |x| = \frac{1 + r y}{y - r} \Rightarrow R_f = (-\infty, -\frac{1}{r}] \cup (r, +\infty)$$

$$y = \frac{1}{x^r - r x} \Rightarrow \frac{1}{x^r - r x + r - r} = \frac{1}{(x - r)^r - r} \Rightarrow y((x - r)^r - r) = 1 \Rightarrow (x - r)^r - r = \frac{1}{y} \Rightarrow (x - r)^r = \frac{1}{y} + r \Rightarrow x - r = \sqrt[r]{\frac{1}{y} + r} \Rightarrow x = \sqrt[r]{\frac{1}{y} + r} + r, \frac{1}{y} + r \geq 0 \Rightarrow \frac{1}{y} \geq -r \Rightarrow y \leq -\frac{1}{r} \Rightarrow y < 0$$

$$\Rightarrow R_f = (-\infty, -\frac{1}{r}] \cup (0, +\infty)$$

$$y = x^r - 9x + r \begin{cases} x_{\min} = \frac{-b}{2a} = \frac{9}{r} = r \\ y_{\min} = 9 - 18 + r = -9 \end{cases} \Rightarrow R_f = [-9, +\infty)$$

$$ب) y = -x^r + 9x + r \begin{cases} x_{\max} = \frac{-b}{2a} = \frac{-9}{-r} = r \\ y_{\max} = -r^r + 18 + r = 9 \end{cases} \Rightarrow R_f = (-\infty, 9]$$

$$\text{الف) } y = \sqrt{x^2 - 4x + 2} \quad \begin{cases} x_{\min} \rightarrow \frac{-b}{2a} = \frac{2}{2} = 1 \\ y_{\min} \rightarrow 9 - 18 + 2 = -7 \end{cases} \rightarrow R_f = \sqrt{[-7, +\infty)} \rightarrow R_f = [-\sqrt{7}, +\infty) \quad \checkmark$$

$$\text{ب) } y = \sqrt{-x^2 + 4x + 11} \quad \begin{cases} x_{\max} \rightarrow \frac{-b}{2a} = \frac{-4}{-2} = 2 \\ y_{\max} \rightarrow -4 + 8 + 11 = 15 \end{cases} \rightarrow R_f = \sqrt{(-\infty, 15]} \rightarrow R_f = [0, \sqrt{15}] \quad \checkmark$$

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$$\text{الف) } y = x^4 + 3x^2 + 2x + 1 \rightarrow R_f = \mathbb{R} \quad \checkmark$$

2

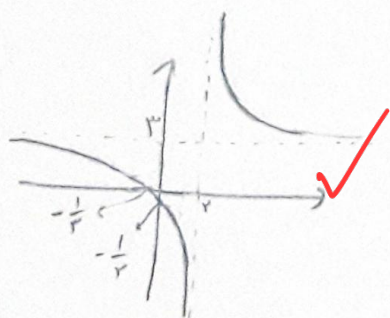
$$\text{ب) } y = \sqrt{x^2 + 4x^2 + 4x + 1} \rightarrow R_f = \sqrt{\mathbb{R}} \Rightarrow R_f = [0, +\infty) \quad \checkmark$$

$$\text{الف) } y = \frac{3x+1}{x-2} \rightarrow R_f = \mathbb{R} - \left\{\frac{1}{2}\right\} \rightarrow R_f = \mathbb{R} - \{2\} \quad \checkmark$$

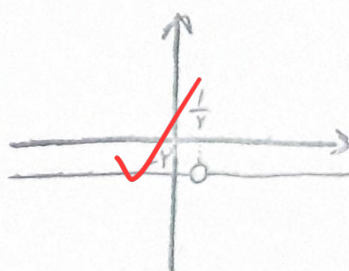
2

$$\text{ب) } y = \sqrt{\frac{4x+1}{x+2}} \rightarrow R_f = \sqrt{\mathbb{R} - \{2\}} \rightarrow R_f = [0, +\infty) - \{2\} \quad \checkmark$$

$$\text{الف) } y = \frac{3x+1}{x-2} \quad \frac{a}{c} = 3 \quad x-2=0 \Rightarrow x=2$$



$$\text{ب) } y = \frac{3x-2}{1-2x} = \frac{2(2x-1)}{-1(2x-1)} = -2$$



2

$$\text{الف) } y = \cos^2 x + \frac{1}{\cos^2 x} \rightarrow a + \frac{1}{a} \rightarrow R_f = [2, +\infty) \quad \checkmark$$

2

$$\text{ب) } y = \sqrt{\frac{x^2+1}{x}} \rightarrow \sqrt{x + \frac{1}{x}} \rightarrow R_f = (-\infty, \sqrt{-1}] \cup [\sqrt{1}, +\infty) \quad \checkmark$$

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