

الف) $y = x^2 - \omega \rightarrow x^2 = y + \omega \rightarrow x = \pm \sqrt{y + \omega} \rightarrow \frac{\omega}{- \phi +} \rightarrow R_f = [-\omega + \infty)$ (2)

ب) $y = x^3 + 1 \rightarrow x^3 = y - 1 \rightarrow x = \sqrt[3]{y - 1} \rightarrow R_f = \mathbb{R}$ (2)

الف) $y = x^2 - \varepsilon x + 4 = \frac{x^2 - \varepsilon x + \varepsilon + 4}{1} \Rightarrow y = (x - \frac{\varepsilon}{2})^2 + 4 - \frac{\varepsilon^2}{4} \rightarrow (x - \frac{\varepsilon}{2})^2 = y - 4 + \frac{\varepsilon^2}{4} \rightarrow x = \pm \sqrt{y - 4 + \frac{\varepsilon^2}{4}} + \frac{\varepsilon}{2} \rightarrow R_f = [\frac{\varepsilon^2}{4} + \infty)$ (2)

ب) $y = x^2 - \omega x + 1 \Rightarrow a > 0 \rightarrow \begin{cases} \min x = \frac{-b}{2a} = \frac{\omega}{2} \\ \min y = \frac{4a}{\omega^2} - \frac{\omega^2}{4} + 1 \end{cases} \rightarrow R_f = [\frac{\omega^2}{4} + \infty)$ (2)

الف) $y = \frac{x^2 + 3}{x^2 - 2} \rightarrow x^2 + 3 = y(x^2 - 2) \rightarrow yx^2 - x^2 = 3 + 2y \Rightarrow x^2(y - 1) = 3 + 2y \rightarrow x = \pm \sqrt{\frac{3 + 2y}{y - 1}}$ (2)

ب) $y = \frac{r|x| + 1}{|x| - \varepsilon} \rightarrow r|x| + 1 = y(|x| - \varepsilon) \rightarrow r|x| - y|x| = -\varepsilon y - 1 \rightarrow |x|(r - y) = -\varepsilon y - 1 \rightarrow |x| = \frac{-\varepsilon y - 1}{r - y} \rightarrow R_f = (-\infty, \frac{1}{\varepsilon}] \cup [2 + \infty)$ (2)

$y = \frac{1}{x^2 - \varepsilon x} \Rightarrow 1 = yx^2 - \varepsilon yx \rightarrow yx^2 - \varepsilon yx - 1 = 0 \rightarrow \Delta = 14y^2 + \varepsilon y$ (1)

$x = \frac{\varepsilon y \pm \sqrt{14y^2 + \varepsilon y}}{2y} \Rightarrow 14y^2 + \varepsilon y \geq 0 \rightarrow 14y^2 \geq -\varepsilon y \rightarrow y \geq -\frac{1}{\varepsilon}$

$\frac{-1}{\varepsilon} \rightarrow R_f = [-\frac{1}{\varepsilon} + \infty)$ (2)

الف) $y = x^2 - 4x + 2, a > 0 \rightarrow \begin{cases} x_{\min} = \frac{-b}{2a} = \frac{4}{2} = 2 \\ y_{\min} = -1 \end{cases} \rightarrow R_f = [-1, +\infty)$ (2)

ب) $y = -x^2 + \varepsilon x + 2, a < 0 \rightarrow \begin{cases} x_{\max} = \frac{-b}{2a} = \frac{-\varepsilon}{-2} = \frac{\varepsilon}{2} \\ y_{\max} = 2 \end{cases} \rightarrow R_f = (-\infty, 2]$ (2)

الف) $y = \sqrt{x^2 - 4x + 4}$ $\Rightarrow$ $\min x = \frac{-b}{2a} = \frac{4}{2} = 2$
 $\min y = -2 \rightarrow R_f = \sqrt{[-2, +\infty)} = [0, +\infty)$ ✓

ب) $y = \sqrt{-x^2 + 4x + 10}$ $\Rightarrow$ $\max x = \frac{-b}{2a} = \frac{-4}{-2} = 2$
 $\max y = 12 \rightarrow R_f = \sqrt{(-\infty, 12]} = [0, \sqrt{12}]$ ✓

الف) $y = x^2 + 2x^2 + 2x + 1 \Rightarrow R_f = \mathbb{R}$ ✓

ب) $y = \sqrt{x^2 + 4x + 4}$ $\Rightarrow R_f = [0, +\infty)$ ✓

الف) $y = \frac{3x+1}{x-2} \Rightarrow R_f = \mathbb{R} - \{3\}$ ✓

ب) $y = \sqrt{\frac{4x+1}{x+2}} \Rightarrow R_f = [0, +\infty) - \{2\}$

الف) $y = \frac{3x+1}{x-2} \Rightarrow$ $\begin{cases} \text{مخرج = 0} \rightarrow x = 2 \\ \text{مقام = 0} \rightarrow y = \frac{a}{c} \rightarrow \frac{3}{-2} = -1.5 \end{cases}$
 $R_f = \mathbb{R} - \{3\}$ ✓

ب) $\frac{4x-2}{1-2x} \Rightarrow y = 2 \Rightarrow R_f = \{-2\}$
 $a \neq \frac{1}{b}$ ✓

الف) $y = \cos^2 x + \frac{1}{\cos^2 x} \rightarrow R_f = [2, +\infty)$ ✓

ب) $y = \sqrt[3]{\frac{x^2+1}{x}} \rightarrow \sqrt[3]{x + \frac{1}{x}} \rightarrow R_f = \sqrt[3]{(-\infty, -1]} \cup [1, +\infty)$

$R_f = (-\infty, \sqrt[3]{-1}] \cup [\sqrt[3]{1}, +\infty)$