

المسألة 1)

الف) $y = x^2 - 8 \rightarrow x^2 = y + 8 \rightarrow x = \pm \sqrt{y + 8} \Rightarrow R_f = [89 + \infty)$

(1)

ب) $y = x^2 + 1 \rightarrow x^2 = y - 1 \rightarrow x = \pm \sqrt{y - 1} \Rightarrow R_f = \mathbb{R}$

الف) $y = x^3 - 8x + 4 \rightarrow y = (x - 2)^3 + 4 \rightarrow (x - 2)^3 = y - 4 \rightarrow x = \pm \sqrt[3]{y - 4} + 2 \Rightarrow R_f = [29 + \infty)$

(2)

ب) $x^3 - 8x + (1 - y) = 0 \rightarrow \Delta \geq 0 \rightarrow \Delta = 48 - 4 + 4y = 44 + 4y \Rightarrow 44 + 4y \geq 0 \rightarrow R_f = [-11 + \infty)$

الف) $x^2 = \frac{y + 3}{y - 1} \rightarrow x = \pm \sqrt{\frac{y + 3}{y - 1}} \rightarrow \frac{-3}{+0} - \frac{1}{-0} +$

(3)

$\rightarrow R_f = (-\infty, \frac{3}{2}] \cup (19 + \infty)$

ب) $|x| = \frac{y + 1}{y - 2} \rightarrow \frac{-1}{+0} - \frac{2}{-0} + \rightarrow R_f = (-\infty, \frac{1}{3}] \cup (29 + \infty)$

(4)

$y = \frac{1}{x^2 - 4} \rightarrow x^2 y - 4xy - 1 = 0 \rightarrow x = \frac{4y \pm \sqrt{16y^2 + 4y}}{2y} \rightarrow \Delta \geq 0$

$\rightarrow 16y^2 + 4y \geq 0 \rightarrow R_f = \mathbb{R}$

الف) $y = x^2 - 4x + 4 \rightarrow \begin{cases} x_{\min} = 2 \\ y_{\min} = -4 \end{cases} \rightarrow R_f = [-49 + \infty)$

(5)

ب) $y = -x^2 + 4x + 4 \rightarrow \begin{cases} x_{\max} = 2 \\ y_{\max} = 4 \end{cases} \rightarrow R_f = (-\infty, 4]$

الف) $y = \sqrt{x^2 - 4x + 4} \rightarrow \begin{cases} x_{\min} = 2 \\ y_{\min} = -4 \end{cases} \rightarrow R_f = \sqrt{[-49 + \infty)} = [09 + \infty)$

(6)

ب) $y = \sqrt{x^2 + 4x + 4} \rightarrow \begin{cases} x_{\max} = 2 \\ y_{\max} = 4 \end{cases} \rightarrow R_f = \sqrt{(-\infty, 4]} = [09 + \infty)$

الف) $R_f = \mathbb{R}$

ب) $R_f = \sqrt{\mathbb{R}} = [09 + \infty)$

(7)

الف) $R_f = \mathbb{R} - \{2\}$

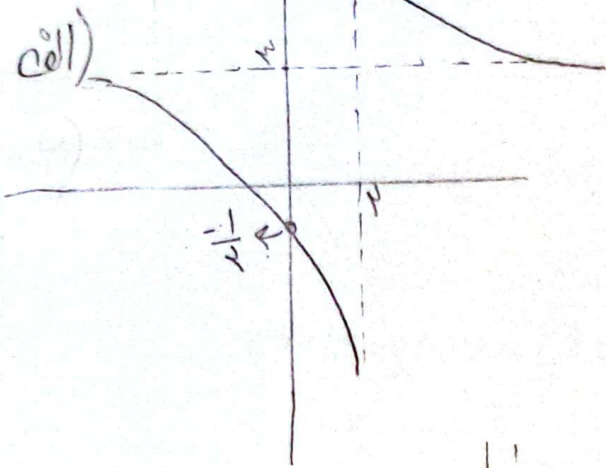
ب) $R_f = [0, +\infty) - \{2\}$

(8)

الحد الأفقي: $x = 2$

الحد العمودي: $y = 3$

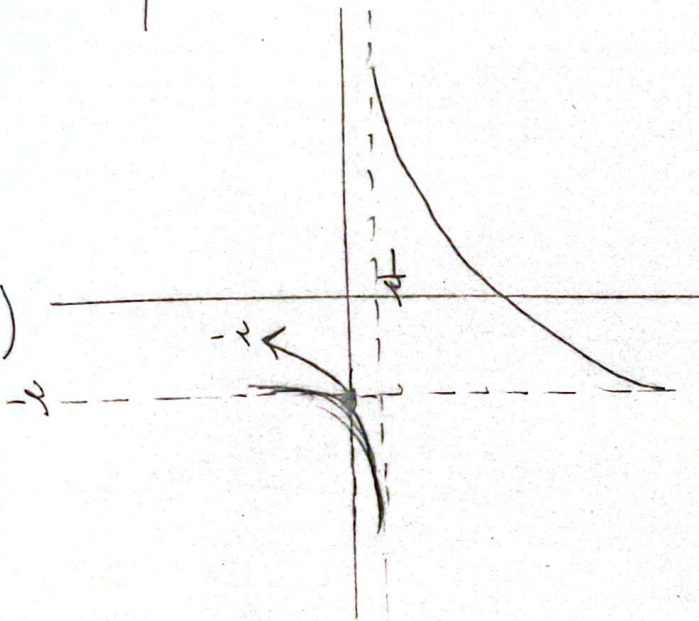
(9)



الحد الأفقي: $x = \frac{1}{2}$

الحد العمودي: $y = -2$

ب)



الف) $R_f = [2, +\infty)$

(10)

ب) $\frac{x^2+1}{x} = x + \frac{1}{x} \rightarrow R_f = \sqrt[3]{[2, +\infty)} = [\sqrt[3]{2}, +\infty)$