

$$\text{الف) } x_{\min} = \frac{-b}{a} = \frac{-(-4)}{1} = 4 \quad y_{\min} = x^2 - 4x + 1 = 16 - 16 + 1 = 1 \Rightarrow R_f = [1, +\infty)$$

$$\text{ب) } x_{\max} = \frac{-b}{a} = \frac{-4}{-1} = 4 \quad y_{\max} = -x^2 + 4x + 1 = -(4)^2 + 4 \times 4 + 1 = 1 \Rightarrow R_f = (-\infty, 1]$$

$$\Rightarrow R_f = (-\infty, +\infty]$$

$$\text{الف) } x_{\min} = \frac{-b}{a} = \frac{-(-4)}{1} = 4 \quad y_{\min} = -1 \Rightarrow R_f = \sqrt{[-1, +\infty)} = [0, +\infty)$$

$$\text{ب) } x_{\max} = \frac{-b}{a} = \frac{-4}{-1} = 4 \quad y_{\max} = 1 \Rightarrow R_f = \sqrt{(-\infty, 1]} = [0, \sqrt{1}]$$

$$\text{الف) } y = x^2 + 2x + 1 \rightarrow \text{بزرگترین توان} \rightarrow R_f = R$$

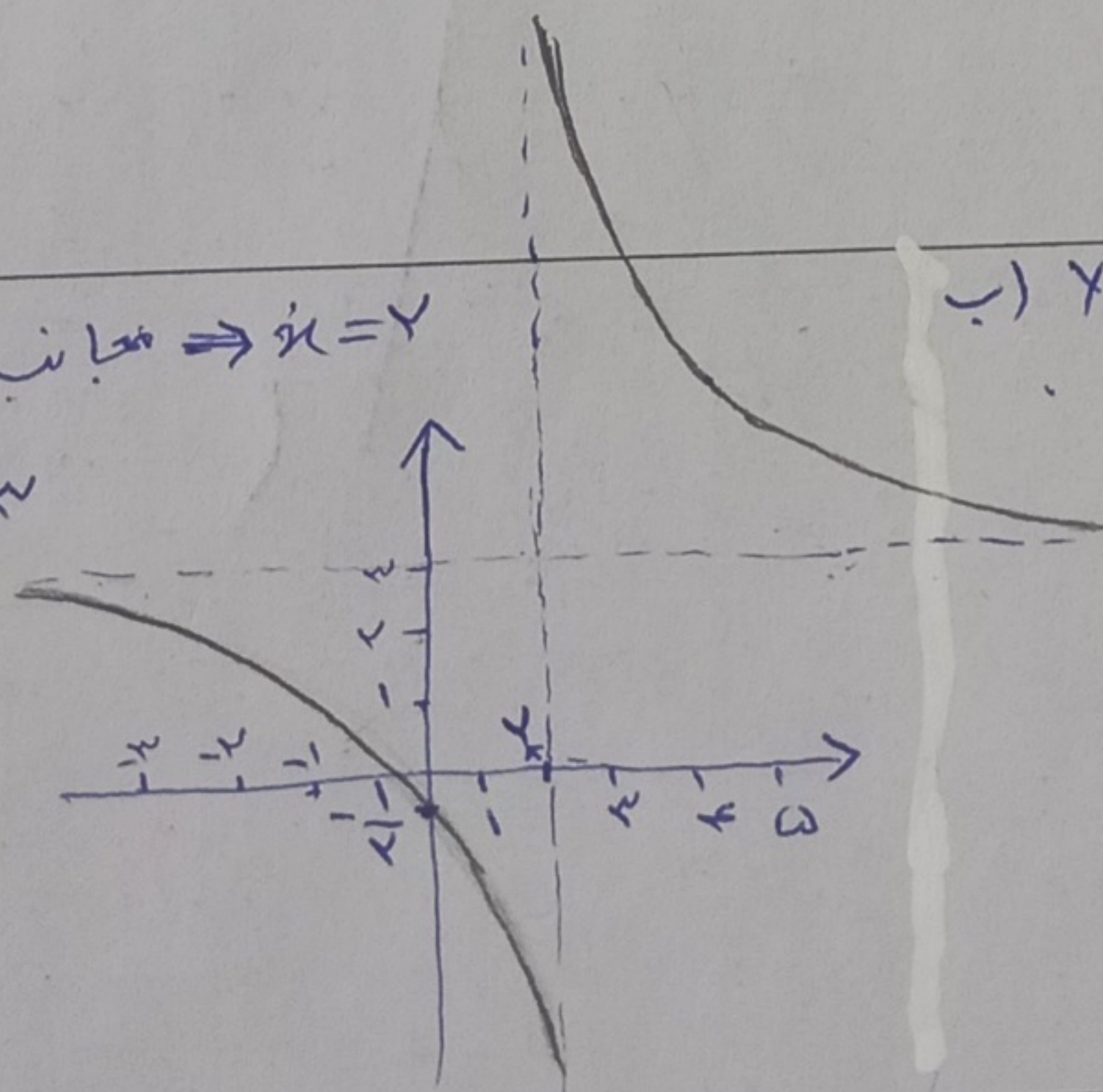
$$\text{ب) } y = \sqrt{x^2 + 4x + 4} \rightarrow \text{بزرگترین توان} \rightarrow R_f = \sqrt{(-\infty, +\infty)} = [0, +\infty)$$

$$\text{الف) } y = \frac{x+1}{x-2} \rightarrow \text{تابع هوگرافیک} \Rightarrow R_f = R - \left\{\frac{a}{c}\right\} = R - \{2\}$$

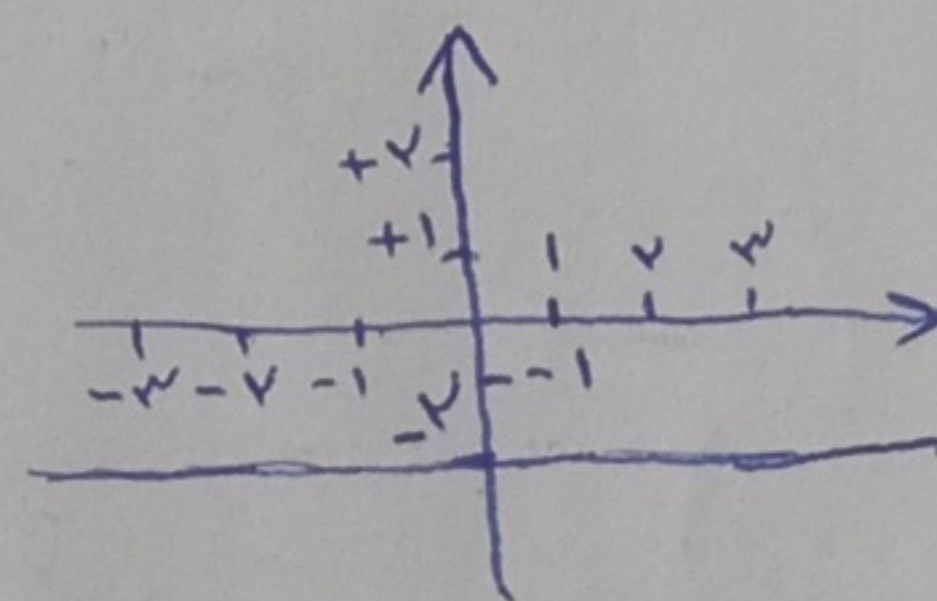
$$\text{ب) } y = \sqrt{\frac{x+1}{x+2}} \rightarrow \text{تابع هوگرافیک} \Rightarrow R_f = \sqrt{(-\infty, +\infty)} - \sqrt{\{2\}} = [0, +\infty) - \{2\}$$

$$\text{الف) } y = \frac{x+1}{x-2} \Rightarrow \text{مجاانب قائم} \Rightarrow x=2$$

$$\text{مجاانب افقی} \rightarrow y = \frac{1}{1} = 1$$



$$\text{ب) } y = \frac{x-2}{1-2x} = -1$$



$$\text{الف) } y = \cos^2 x + \frac{1}{\cos^2 x} \xrightarrow{\cos^2 x > 0} R_f = [2, +\infty)$$

$$\text{ب) } y = \sqrt[3]{\frac{x^2+1}{x}} = \sqrt[3]{x + \frac{1}{x}} \Rightarrow R_f = (-\infty, -2] \cup [2, +\infty)$$

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چون فو به 3 محدودیت ندارد

الف) $y = x^2 - 5 \Rightarrow x^2 = y + 5 \Rightarrow y + 5 \geq 0 \Rightarrow R_f = [-5, +\infty)$
 $x = \pm \sqrt{y + 5}$

ب) $y = x^3 + 1 \Rightarrow x^3 = y - 1 \Rightarrow x = \sqrt[3]{y - 1} \Rightarrow R_f = \mathbb{R}$

الف) $y = x^2 - 4x + 4 + 2 \Rightarrow y = (x - 2)^2 + 2 \Rightarrow y - 2 = (x - 2)^2$

$x - 2 = \pm \sqrt{y - 2} \Rightarrow R_f = [2, +\infty)$

ب) $y = x^2 - 4x + 1 + 4 + 4 - 4 + 2 = (x - 2)^2 - 4 + 2 \Rightarrow$

$\Rightarrow y + 4 = (x - 2)^2 \Rightarrow x - 2 = \pm \sqrt{y + 4} \Rightarrow$

$\Rightarrow R_f = [-4, +\infty)$

الف) $y = \frac{x^2 + 3}{x^2 - 1} \Rightarrow y(x^2 - 1) = x^2 + 3 \Rightarrow -yx^2 + x^2 + y + 3 = 0$

$\Rightarrow x^2(1 - y) + y + 3 = 0 \Rightarrow x = \frac{-b \pm \sqrt{\Delta}}{a} = \frac{0 \pm \sqrt{0^2 - 4(1 - y)(y + 3)}}{1 - y}$

$\Rightarrow -4(1 - y)(y + 3) \geq 0 \Rightarrow \frac{-\frac{3}{y}}{+\infty - \frac{1}{y} +} \geq 0 \Rightarrow 1 - y \neq 0 \Rightarrow R_f = (-\infty, -\frac{3}{y}] \cup (1, +\infty)$

ب) $y|x| - 4y = y|x| + 1 \Rightarrow -y|x| + y|x| = 4y + 1$

$\Rightarrow |x|(y - 4) = 4y + 1 \Rightarrow |x| = \frac{4y + 1}{y - 4} \Rightarrow R_f = (-\infty, -\frac{1}{4}] \cup (4, +\infty)$

$y = \frac{1}{x^2 - 4x} \Rightarrow y(x^2 - 4x) = 1 \Rightarrow yx^2 - 4yx - 1 = 0$

$x = \frac{-b \pm \sqrt{\Delta}}{a} = \frac{4 \pm \sqrt{16y^2 - 4y(-1)}}{y} \Rightarrow \frac{16y^2 + 4y}{y^2} \geq 0 \Rightarrow \frac{-\frac{1}{4}}{+\infty - \frac{1}{y} +} \geq 0$

$\Rightarrow R_f = (-\infty, -\frac{1}{4}] \cup (0, +\infty)$