

الف) $x = \frac{b}{a} = \frac{-(-4)}{1} = 4$ $y_{min} = x^2 - 4x + 1 = 16 - 16 + 1 = 1 \Rightarrow R_f = [1, +\infty)$ ✓

ب) $x = \frac{b}{a} = \frac{-4}{-1} = 4$ $y_{max} = -x^2 + 4x + 1 = -(4)^2 + 4 \times 4 + 1 = 1$ ✓
 $\Rightarrow R_f = (-\infty, 1]$ ✓

الف) $x_{min} = \frac{-b}{2a} = \frac{-(-4)}{2} = 2$ $y_{min} = -4 \Rightarrow R_f = \sqrt{[-4, +\infty)} = [0, +\infty)$ ✓

ب) $x_{max} = \frac{-b}{2a} = \frac{-4}{-2} = 2$ $y_{max} = 14 \Rightarrow R_f = \sqrt{(-\infty, 14]} = [0, \sqrt{14}]$ ✓

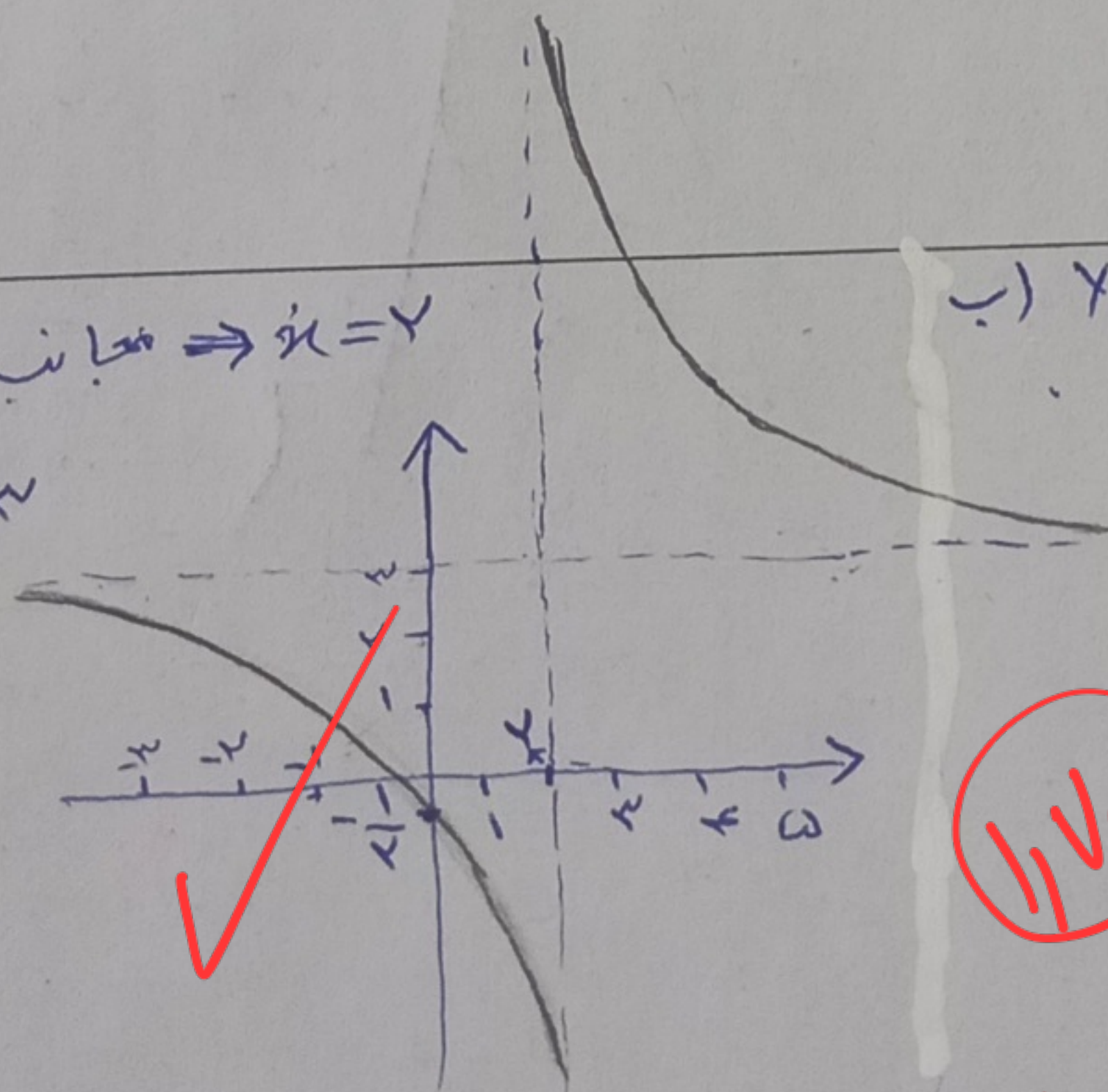
الف) $y = x^2 + 4x^2 + 2x + 1 \rightarrow$ بزرگی بین توانی $\rightarrow R_f = R$ ✓

ب) $y = \sqrt{x^2 + 4x^2 + 2x + 1} \rightarrow$ بزرگی بین توانی $\rightarrow R_f = \sqrt{(-\infty, +\infty)} = [0, +\infty)$ ✓

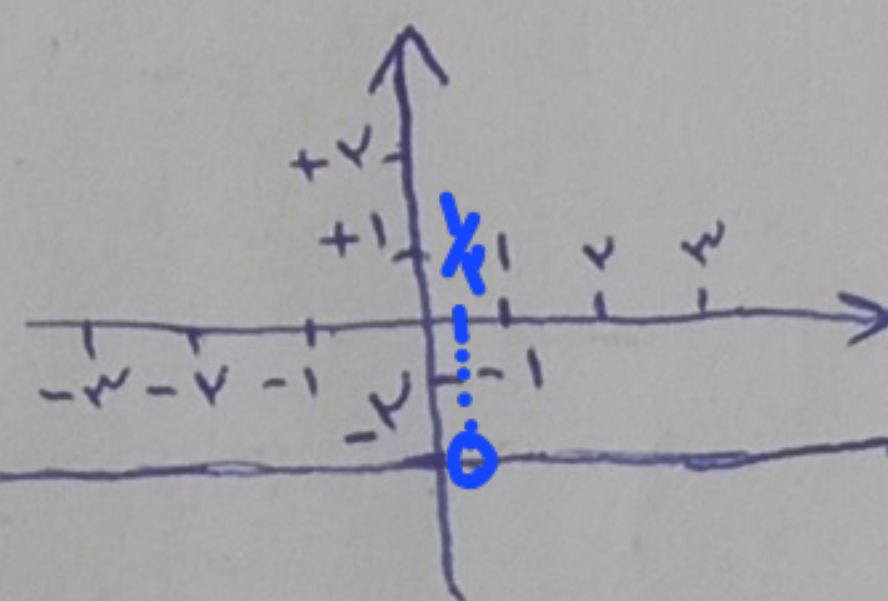
الف) $y = \frac{x+1}{x-2} \rightarrow$ تابع هوگرافیک $\Rightarrow R_f = R - \{\frac{a}{c}\} = R - \{2\}$ ✓

ب) $y = \sqrt{\frac{x+1}{x+3}} \rightarrow$ تابع هوگرافیک $\Rightarrow R_f = \sqrt{(-\infty, +\infty) - \sqrt{\frac{1}{3}}} = [0, +\infty) - \{\frac{\sqrt{3}}{3}\}$ ✓

الف) $y = \frac{x+1}{x-2}$ $\Rightarrow$ معادله قائم $x=2$
 معادله افقی $\rightarrow y = \frac{2}{1} = 2$



ب) $y = \frac{x-2}{1-2x} = -\frac{1}{2}$



الف) $y = \cos^2 x + \frac{1}{\cos^2 x}$ $\xrightarrow{\cos^2 x > 0} R_f = [2, +\infty)$ ✓

ب) $y = \sqrt[3]{\frac{x^2+1}{x}} = \sqrt[3]{x + \frac{1}{x}} \Rightarrow R_f = \sqrt[3]{(-\infty, -2] \cup [2, +\infty)}$

$R_f = (-\infty, \sqrt[3]{-2}] \cup [\sqrt[3]{2}, +\infty)$

الف) $y = x^2 - 5 \Rightarrow x^2 = y + 5 \Rightarrow x = \pm \sqrt{y+5} \Rightarrow R_f = [-5, +\infty)$

ب) $y = x^3 + 1 \Rightarrow x^3 = y - 1 \Rightarrow x = \sqrt[3]{y-1} \Rightarrow R_f = \mathbb{R}$

الف) $y = x^2 - 4x + 4 + y \Rightarrow y = (x-2)^2 + y \Rightarrow y - y = (x-2)^2 \Rightarrow x-2 = \pm \sqrt{y-y} \Rightarrow R_f = [y, +\infty)$

ب) $y = x^2 - 4x + 1 + 4x - 4 = (x-2)^2 - 3 \Rightarrow y + 3 = (x-2)^2 \Rightarrow x-2 = \pm \sqrt{y+3} \Rightarrow R_f = [-3, +\infty)$

الف) $y = \frac{x^2 + 3}{x^2 - 1} \Rightarrow y(x^2 - 1) = x^2 + 3 \Rightarrow -yx^2 + x^2 + yx + 3 = 0$

$\Rightarrow x^2(1-y) + yx + 3 = 0 \Rightarrow x = \frac{-y \pm \sqrt{y^2 - 4(1-y)(y+3)}}{2(1-y)}$
 $\Rightarrow -4(1-y)(y+3) \geq 0 \Rightarrow y \in (-\infty, -\frac{3}{4}] \cup (1, +\infty)$

ب) $y|x| - 4y = |x| + 1 \Rightarrow -y|x| + y|x| = 4y + 1$
 $\Rightarrow |x|(y-4) = 4y + 1 \Rightarrow |x| = \frac{4y+1}{y-4} \Rightarrow R_f = (-\infty, -\frac{1}{4}] \cup (4, +\infty)$

$y = \frac{1}{x^2 - 4x} \Rightarrow yx^2 - 4yx = 1 \Rightarrow yx^2 - 4yx - 1 = 0$

$x = \frac{-b \pm \sqrt{\Delta}}{a} = \frac{4 \pm \sqrt{16y^2 - 4y(-1)}}{y} \Rightarrow 16y^2 + 4y \geq 0 \Rightarrow y \in (-\infty, -\frac{1}{4}] \cup (0, +\infty)$