

الف) $y = x^2 - 5 \Rightarrow x^2 = y + 5 \Rightarrow x = \pm \sqrt{y + 5} \Rightarrow y \geq -5 \Rightarrow R(f) = [-5, +\infty)$
 ب) $y = x^3 + 1 \Rightarrow x^3 = y - 1 \Rightarrow x = \sqrt[3]{y - 1} \Rightarrow R(f) = \mathbb{R}$

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الف) $y = x^2 - 4x + 4 = (x - 2)^2 = y - 2 \Rightarrow x - 2 = \pm \sqrt{y - 2} \Rightarrow x = 2 \pm \sqrt{y - 2} \Rightarrow y \geq 2 \Rightarrow R(f) = [2, +\infty)$

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ب) $y = x^2 - 5x + 1 \Rightarrow y = (x - \frac{5}{2})^2 - \frac{25}{4} + 1 = (x - \frac{5}{2})^2 - \frac{21}{4} \Rightarrow (x - \frac{5}{2})^2 = y + \frac{21}{4} \Rightarrow y + \frac{21}{4} \geq 0 \Rightarrow y \geq -\frac{21}{4} \Rightarrow R(f) = [-\frac{21}{4}, +\infty)$

الف) $y = \frac{x^2 + 3}{x^2 - 2} \Rightarrow y(x^2 - 2) = x^2 + 3 \Rightarrow yx^2 - 2y = x^2 + 3 \Rightarrow yx^2 - x^2 = 2y + 3 \Rightarrow x^2(y - 1) = 2y + 3 \Rightarrow x^2 = \frac{2y + 3}{y - 1} \Rightarrow x = \pm \sqrt{\frac{2y + 3}{y - 1}} \Rightarrow \frac{2y + 3}{y - 1} \geq 0 \Rightarrow R(f) = (-\infty, -\frac{3}{2}] \cup (1, +\infty)$

ب) $y = \frac{x^2 + 1}{|x| - 1} \Rightarrow y(|x| - 1) = x^2 + 1 \Rightarrow y|x| - y = x^2 + 1 \Rightarrow |x| = \frac{x^2 + 1 + y}{y} \Rightarrow |x| \geq 0 \Rightarrow \frac{x^2 + 1 + y}{y} \geq 0 \Rightarrow R(f) = (-\infty, -\frac{1}{2}] \cup (1, +\infty)$

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الف) $y = \frac{1}{x^2 - 4x} = \frac{1}{(x - 2)^2 - 4} \Rightarrow D_f = (-\infty, -2) \cup (-2, 2) \cup (2, +\infty) \Rightarrow R = \frac{xy \pm \sqrt{xy^2 + 4y}}{xy}$

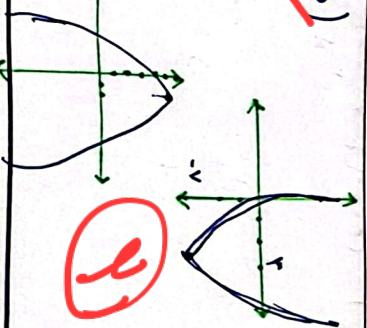
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$f = (-\infty, -\frac{1}{4}] \cup (0, +\infty)$

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الف) $y = x^2 - 4x + 2 \Rightarrow x^2 - 4x + 2 = y \Rightarrow x^2 - 4x + 4 - 2 = y \Rightarrow (x - 2)^2 = y + 2 \Rightarrow x - 2 = \pm \sqrt{y + 2} \Rightarrow x = 2 \pm \sqrt{y + 2} \Rightarrow y \geq -2 \Rightarrow R(f) = [-2, +\infty)$
 ب) $y = -x^2 + 4x + 2 \Rightarrow -x^2 + 4x + 2 = y \Rightarrow -x^2 + 4x = y - 2 \Rightarrow x^2 - 4x = 2 - y \Rightarrow (x - 2)^2 = 4 - y \Rightarrow x - 2 = \pm \sqrt{4 - y} \Rightarrow x = 2 \pm \sqrt{4 - y} \Rightarrow y \leq 4 \Rightarrow R(f) = (-\infty, 4]$

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الف) $y = \sqrt{u^2 - 4u + 1}$ $\rightarrow y = \frac{-\Delta}{2a} = \frac{-(4u-1)}{2} = -u \Rightarrow y \in [-u, +\infty)$

$\Rightarrow R(f) = [0, +\infty)$ ✓

ب) $y = \sqrt{-u^2 + 4u + 1}$ $\rightarrow y = \frac{-\Delta}{2a} = \frac{-(-4u-1)}{-2} = 1 \Rightarrow y \in (-\infty, 1] \Rightarrow$

$R(f) = [0, \sqrt{14}]$ ✓ 1

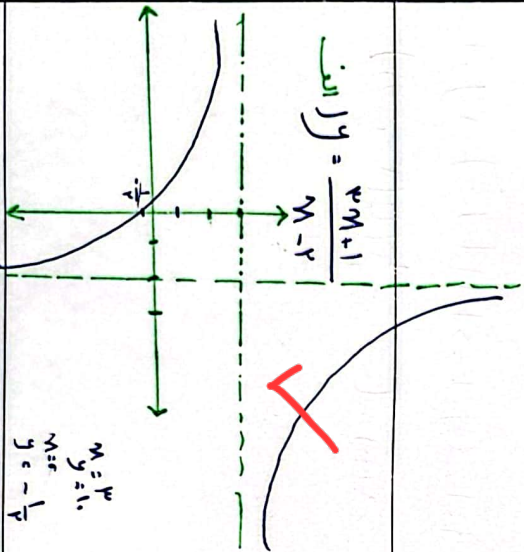
الف) $y = u + 3u^2 + 2u + 1$ $R(f) = \mathbb{R}$ ✓

ب) $y = \sqrt{u^2 + 4u + 1} \Rightarrow \sqrt{14}$ $\Rightarrow R(u) = [0, +\infty)$ 1

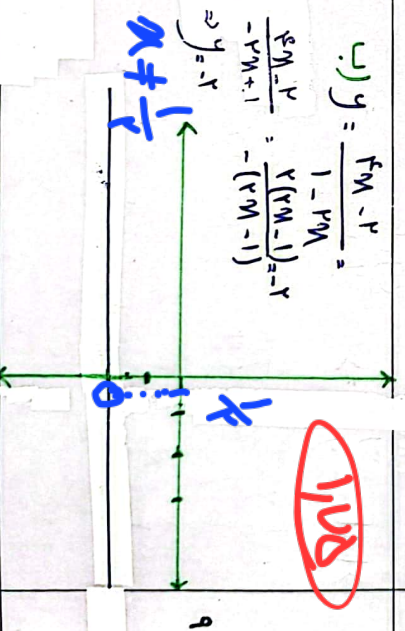
الف) $y = \frac{3u+1}{u-2} \Rightarrow R(f) = \mathbb{R} - \{3\}$ ✓

ب) $y = \sqrt{\frac{4u+1}{u+2}} \Rightarrow \sqrt{14 - \{3\}}$ $\Rightarrow R(f) = [0, +\infty) - \{3\}$ 1

الف) $y = \frac{3u+1}{u-2}$



ب) $y = \frac{4u-2}{-2u+1} = \frac{1-3u}{-(2u-1)}$



الف) $y = \cos^2 u + \frac{1}{\cos^2 u}$

$\Rightarrow \{y \mid y \geq 2\} \Rightarrow R(f) = [2, +\infty)$

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ب) $y = \int \frac{3}{u^2+1}$

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$\frac{3}{u} + \frac{1}{u} = u + \frac{1}{u}$

$\Rightarrow R(f) = \sqrt{(-\infty, -1]} \cup [1, +\infty)$

$R_f = (-\infty, \sqrt{-1}] \cup [\sqrt{1}, +\infty)$