

الف) $f(x) = \sqrt{x|x|}$, $g(x) = x \rightarrow x|x| \geq 0 \rightarrow x \geq 0 \rightarrow f(x) = g(x)$
 $\rightarrow D_f = \mathbb{R}$

ب) $f(x) = \frac{(x+3)(x+1) + x + 1}{x^2 + 2x + 1}$, $g(x) = 1 \rightarrow D_f = \mathbb{R} \rightarrow D_g = \mathbb{R} \rightarrow f(x) = g(x)$

ج) $f(x) = \frac{2 \sin^2 x - 4 \sin x}{2 \sin x - 2}$, $g(x) = 2 \sin x \rightarrow -2 \leq \sin x \leq 2 \rightarrow D_g = \mathbb{R} \rightarrow f(x) = g(x)$
 $\rightarrow D_f = \mathbb{R}$

د) $f(x) = \frac{x}{|x|}$, $g(x) = \frac{|x|}{x} \rightarrow x \neq 0 \rightarrow |x| \neq 0 \rightarrow x \neq 0 \rightarrow f(x) = g(x)$

الف) $f(x) = \left[\frac{x^2}{x^2+1} \right]$, $g(x) = [x - [x]] \rightarrow f(x) = g(x) \rightarrow D_f = \mathbb{R} \rightarrow D_g = \mathbb{R}$
 ب) $f(x) = \frac{1}{[x]}$, $g(x) = \frac{1}{[x]}$
 $[x] \neq 0 \rightarrow x \in (0, 1) \cup (1, 2) \cup \dots$
 $D_f = (-\infty, 0) \cup (1, +\infty)$
 ج) $f(x) = \frac{1}{\sqrt{x-1}}$, $g(x) = \frac{1}{f(x)}$
 $x-1 > 0 \rightarrow x > 1 \rightarrow |x-1| > 0 \rightarrow D_g = \emptyset$
 د) $f(x) = \begin{cases} \frac{x^2-1}{x} & ; x \neq 1 \\ 0 & ; x = 1 \end{cases}$, $g(x) = x^2 + x \rightarrow f(x) \neq g(x)$
 $x-1 \neq 0 \rightarrow x \neq 1 \rightarrow D_f = \mathbb{R} \rightarrow D_g = \mathbb{R} \rightarrow f(x) = g(x)$

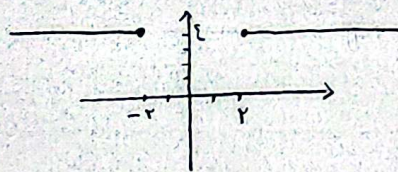
$f(x) + g(x) = x^2 + x + 1 \rightarrow x^2 - x + 1 + g(x) + g(x) = x^2 + x + 1 \rightarrow 2g(x) = 2x \rightarrow g(x) = x$

$f(x) - g(x) = x^2 - x + 1 \rightarrow f(x) = x^2 - x + 1 + g(x)$

$f(x) = x^2 + 1$

$f^2(x) - g^2(x) = (x^2+1)^2 - x^2 = x^4 + 1 + 2x^2 - x^2 = x^4 + x^2 + 1$

$f(x) = x - \sqrt{x^2-4}$
 $g(x) = x + \sqrt{x^2-4}$
 $f \cdot g \rightarrow (x - \sqrt{x^2-4})(x + \sqrt{x^2-4}) = x^2 - x^2 + 4 = 4$



$x \geq 2$
 $x \leq -2$

$\Rightarrow D_{f \cdot g} = (-\infty, -2] \cup [2, +\infty)$

$f(x) = \frac{ax+b}{x^2-mx+n} \rightarrow -m = \frac{a}{r} + 1 = -\frac{r}{r} \rightarrow m = \frac{r}{r}$
 $n = -\frac{b}{r} \times 1 = -\frac{b}{r}$

$g(x) = \frac{x-b}{r}$

$\frac{ax^2-bx-a}{x^2-rx-r} \rightarrow x^2-r-10 = (x-5)(x+r) = 0 \rightarrow 5, -r$

$\frac{x}{r} - \frac{b}{r} = \frac{ax+b}{x^2-mx+n} \rightarrow -\frac{b}{r} = r \rightarrow b = -r$

$am - bn = (\frac{1}{r} \times \frac{r}{r}) - ((-r) \times (-\frac{b}{r}))$
 $= \frac{r}{r} - 10 = -9, 15$

$\frac{x^2-r}{x^2-rx-r} \rightarrow a = \frac{1}{r}$

$$f(x) = \frac{bx+2}{\lambda x+b} \rightarrow \lambda x+b \neq 0 \rightarrow x \neq -\frac{b}{\lambda}$$

$$g(x) = c \rightarrow D_g = \mathbb{R} - \{c\}$$

چون دایره دایره برابر است بنابراین $-\frac{b}{\lambda}$ برابر a می شود
برای برابری تابع باید صورت $(-\frac{b}{\lambda}) + 2 = c$ برابر صفر باشد

$$a = -\frac{b}{\lambda} \rightarrow (b(-\frac{b}{\lambda}) + 2 = 0 \rightarrow \frac{+b^2}{\lambda} = -2 \rightarrow b = \pm 2, a = \mp \frac{1}{2}, c = \pm \frac{1}{2}$$

$$\frac{ab}{c} = \frac{\frac{1}{2} \times \frac{1}{2}}{\frac{1}{2}} = \frac{1}{2}$$

$$f^{-1} = \{(-1, 3), (2, 7), (3, -5), (0, 0)\} \rightarrow f = \{(-1, 2), (2, 4), (3, -2), (0, \frac{1}{2})\}$$

$$g = \{(2, 4), (3, 6), (-1, -2), (5, 10)\}$$

$$\{(-1, -2), (2, 8), (3, 4)\}$$

$$\{1, 4, 3\} = \text{برون تابع} \leftarrow \{(2, 1), (-1, 4), (3, 3)\}$$

$$f = \{(2, a), (-3, 3a-2b), (c, d), (2, d)\} \rightarrow a = d = -1$$

$$g = \{(2, -1), (-3, 1), (a-3b, d)\} \rightarrow a-3b = c \xrightarrow{\frac{a=-1}{b=-2}} -1-(-6) = 5$$

$$\rightarrow 3a-2b = 1 \xrightarrow{\frac{a=-1}{b=-2}} -2 = 1 \rightarrow b = -\frac{1}{2}$$

$$a = -1, b = -2, c = 5, d = -1$$

$$d+c = -1+5 = 4$$

$$f(x) = \sqrt{-x^2+x-m} \rightarrow \text{Max}$$

$$D_g = D_f = a$$

$$g(x) = \{(a, b)\}$$

$$b^2 - 4ac \rightarrow 1 - 4m = 0 \rightarrow m = \frac{1}{4}$$

$$f(x) = \sqrt{-x^2+x-\frac{1}{4}}$$

$$\text{Max Ext} \left| \begin{array}{l} \frac{1}{4} \\ 0 \end{array} \right| \Rightarrow a = \frac{1}{4}, b = 0 \rightarrow a+b = \frac{1}{4} + 0 = \frac{1}{4}$$

$$f(x) = \frac{2x+a}{x^2+4x+b}$$

با توجه به اینکه صفت g یک مثل $x=c$ می باشد بنابراین خروج صفت f باید برابر $x=c$ باشد.

$$g(x) = \frac{2}{x-c}$$

$$\frac{2x+a}{(x-c)^2} = \frac{2x+a}{x^2-2cx+c^2} = \frac{2x+a}{x^2+4x+b}$$

$$\rightarrow b = c^2 \rightarrow b = 9$$

$$-2c = 4 \rightarrow c = -2$$

$$a+b+c = 4+9-2 = 11$$

$$\xrightarrow{x=1} \frac{2 \times 1 + a}{1+4+9} = \frac{2}{1+9}$$

$$\Rightarrow 2 = 1+a \rightarrow a = 1$$