

نام و نام خانوادگی: علی حسینیان پاسخنامه تشریحی تکلیف شماره ۱۸ کلاس: پسر

۱) $g(x) = 1 \Rightarrow Dg = \mathbb{R}$ $Dg = \mathbb{R}$ $f(x) = \frac{x^2 + \Delta x + \nabla}{x^2 + \Delta x + \nabla} = 1$ $\Rightarrow$ با هم برابرند

۲) $g(x) = 2 \sin x \Rightarrow Dg = \mathbb{R}$ $Dg = \mathbb{R}$ $f(x) = \frac{2 \sin x (2 \sin x - 1)}{2 \sin x - 1} = 2 \sin x$ $\Rightarrow$ با هم برابرند

۳) $f(x) = \frac{x}{1+x}$ $g(x) = \frac{1}{x}$ $\Rightarrow$ $\frac{1}{x} = \frac{x}{1+x} \Rightarrow 1+x = x^2 \Rightarrow x^2 - x - 1 = 0$ $\Rightarrow$ $x = \frac{1 \pm \sqrt{5}}{2}$ $\Rightarrow$ $Df = \mathbb{R} \setminus \{-\frac{1}{2}\}$ $Dg = \mathbb{R} \setminus \{0\}$ $\Rightarrow$ $D_{f \times g} = \mathbb{R} \setminus \{0, -\frac{1}{2}\}$

۴) $g(x) = x \Rightarrow Dg = \mathbb{R}$ $f(x) = \sqrt{x|x|} \Rightarrow x|x| \geq 0 \Rightarrow Df = [0, +\infty)$

الف) $Df = \mathbb{R}$ $Dg = \mathbb{R}$ $f(x) = x - [x] \Rightarrow 0 \leq x - [x] < 1 \Rightarrow [x - [x]] = 0$ $\Rightarrow$ $Df = \mathbb{R}$ $Dg = \mathbb{R}$ $\Rightarrow$ با هم برابرند

ب) $x - |x| > 0 \Rightarrow$ $\begin{cases} \text{if } x \geq 0 \Rightarrow x - |x| = 0 \\ \text{if } x < 0 \Rightarrow x - |x| = x - (-x) = 2x < 0 \end{cases} \Rightarrow Df = \emptyset$

ج) $|x| - x > 0 \Rightarrow$ $\begin{cases} \text{if } x \geq 0 \Rightarrow |x| - x = 0 \\ \text{if } x < 0 \Rightarrow |x| - x = -x - x = -2x > 0 \end{cases} \Rightarrow Dg = (-\infty, 0)$

د) $Df = \mathbb{R}$ $Dg = \mathbb{R}$ $f(x) = \frac{x(x-1)(x+1)}{x-1} = x^2 + x = g(x)$ $\Rightarrow$ با هم برابرند

$f(x) + g(x) = x^2 + x + 1$ $f(x) - g(x) = x^2 - x + 1$ $\Rightarrow$ $2f(x) = 2x^2 + 2 \Rightarrow f(x) = x^2 + 1$ $2g(x) = 2x \Rightarrow g(x) = x$

$f(x) - g(x) = (x^2 + 1) - x = x^2 - x + 1$

$Df \rightarrow x^2 - 4 > 0 \Rightarrow x^2 > 4 \Rightarrow x > 2 \vee x < -2 \Rightarrow Df = (2, +\infty) \cup (-\infty, -2)$

$Dg \rightarrow x^2 - 4 > 0 \Rightarrow x^2 > 4 \Rightarrow x > 2 \vee x < -2 \Rightarrow Dg = (2, +\infty) \cup (-\infty, -2)$

$D_{f \times g} = Df \cap Dg = (2, +\infty) \cup (-\infty, -2)$

سؤال: خود تابع $f \times g$ دامنه تابع $f \times g$ ؟

$2x^2 - 3x - 8 = 2(x^2 - \frac{3}{2}x - \frac{8}{2}) \rightarrow \frac{ax+b}{x^2-mx+n} = \frac{x-b}{x^2-\frac{3}{2}x-\frac{8}{2}}$

$x^2 - mx + n = x^2 - \frac{3}{2}x - \frac{8}{2}$ $\Rightarrow$ $-m = -\frac{3}{2} \Rightarrow m = \frac{3}{2}$ $n = -\frac{8}{2} = -4$

$ax+b = 2(x-b) \Rightarrow ax+b = 2x-2b \Rightarrow a = 2$ $b = -2b \Rightarrow b = 0$

$am - bn = 3 - \frac{8}{2} = \frac{1}{2}$

$$f(x) \rightarrow \wedge x + b \neq 0 \Rightarrow \wedge x \neq -b \Rightarrow x \neq -\frac{b}{\lambda} \Rightarrow D_f = \mathbb{R} - \left\{ -\frac{b}{\lambda} \right\}, D_f = D_g \Rightarrow a = -\frac{b}{\lambda}$$

$$\frac{b}{\lambda} = \frac{x}{b} \Rightarrow b^2 = 17 \Rightarrow b = \begin{cases} \sqrt{17} \rightarrow a = -\frac{1}{\sqrt{17}}, c = \frac{1}{\sqrt{17}} \Rightarrow \frac{ab}{c} = +1 \\ -\sqrt{17} \rightarrow a = \frac{1}{\sqrt{17}}, c = -\frac{1}{\sqrt{17}} \Rightarrow \frac{ab}{c} = -1 \end{cases}$$

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$$2f(-1) - 1 = 3 \Rightarrow 2f(-1) = 4 \Rightarrow f(-1) = 2$$

$$D_f = \{-1, 0, 2, 3\}$$

$$2f(2) - 1 = 1 \Rightarrow 2f(2) = 2 \Rightarrow f(2) = 1$$

$$D_g = \{-1, 2, 3, 4\}$$

$$2f(3) - 1 = -2 \Rightarrow 2f(3) = -1 \Rightarrow f(3) = -\frac{1}{2}$$

$$D_{f+g} = D_f \cap D_g = \{-1, 2, 3\}$$

$$2f(0) - 1 = 0 \Rightarrow 2f(0) = 1 \Rightarrow f(0) = \frac{1}{2}$$

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$$\text{if } x = -1 \rightarrow f(-1) = 2, g(-1) = 4 \quad \text{if } x = 2 \rightarrow f(2) = 1, g(2) = 4 \quad \text{if } x = 3 \rightarrow f(3) = -\frac{1}{2}, g(3) = 4$$

$$\frac{2g(-1)}{(f(-1)+g(-1))} = \frac{-1}{-1} = 1$$

$$\frac{2g(2)}{(f(2)+g(2))} = \frac{1}{1} = 1$$

$$\frac{2g(3)}{(f(3)+g(3))} = \frac{12}{\frac{1}{2}} = 24 \Rightarrow \text{جواب} = \{1, 2, 24\}$$

$$d = a = -1 \quad a - 3b = c \quad 2a - 2b = 1$$

$$-1 + 7 = 2$$

$$-2 - 2b = 1$$

$$\Rightarrow d + c = 4$$

$$c = 2$$

$$-2b = 3 \rightarrow b = -\frac{3}{2}$$

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$$D_g = \{a\} \quad -x^2 + x - m \geq 0 \quad \Delta = 0 \Rightarrow b^2 - 4ac = 0$$

$$1^2 - 4(-1)(-m) = 0 \Rightarrow 1 - 4m = 0 \Rightarrow m = \frac{1}{4} \quad a = \frac{-b}{2a} = \frac{-1}{-2} = \frac{1}{2} = a$$

$$g(a) = b \rightarrow f\left(\frac{1}{a}\right) = b \rightarrow f\left(\frac{1}{a}\right) = \sqrt{-\left(\frac{1}{a}\right)^2 + \frac{1}{a} - \frac{1}{a}} = 0$$

$$f\left(\frac{1}{a}\right) = 0 = b \Rightarrow a + b = 0 + \frac{1}{a} = \frac{1}{a}$$

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$$x - c \neq 0 \Rightarrow x \neq c \quad D_g = \mathbb{R} - \{c\} \Rightarrow x^2 + 9x + b \xrightarrow{=0} c \Rightarrow \Delta = 0 = 9^2 - 4 \times 1 \times b \Rightarrow b = 9$$

$$\Rightarrow x^2 + 9x + 9 = (x+3)(x+3) \Rightarrow c = -3$$

$$f(x) = f(g) \Rightarrow \frac{2x+a}{(x+3)^2} = \frac{2}{(x+3)} \Rightarrow 2x+a = 2x+6 \Rightarrow a = 6$$

$$\Rightarrow a + b + c = 12$$

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