

الف) $f(x) = \sqrt{x|x|}$ و $g(x) = x$

۱) $f \Rightarrow x|x| \geq 0 \Rightarrow x \geq 0$ مساوی نیستند
 $D_f = [0, +\infty)$

$D_g = \mathbb{R}$

ب) $f(x) = \frac{(x+3)(x+1)x + 4}{x^2 + 2x + 3}$ و $g(x) = 1$

۱) $f \Rightarrow x^2 + 2x + 3 \neq 0$

$\Delta = b^2 - 4ac = 2^2 - 4 \cdot 1 \cdot 3 = -8 < 0 \Rightarrow$ ریشه ندارد $D_f = \mathbb{R}$

$D_g = \mathbb{R}$

$f(x) \Rightarrow$ if $x = 0, 1, 2 \Rightarrow f(x) = 1$ دو تابع مساوی هستند ✓

ج) $f(x) = \frac{2 \sin^2 x - 2 \sin x}{2 \sin x - 3}$ و $g(x) = 2 \sin x$

۱) $f \Rightarrow 2 \sin x - 3 \neq 0$

$\Rightarrow D_f = \mathbb{R}$

همواره برقرار ✓ $\sin x \neq 1.5$

$D_g = \mathbb{R}$

دو تابع مساوی هستند ✓

د) $f(x) = \frac{x}{|x|}$ و $g(x) = \frac{|x|}{x}$

$D_f \Rightarrow |x| \neq 0 \Rightarrow x \neq 0 \Rightarrow D_f = \mathbb{R} - \{0\}$

$D_g \Rightarrow x \neq 0 \Rightarrow D_g = \mathbb{R} - \{0\}$

if $x = 1, -1, 2, -2, \dots \Rightarrow f(x) = g(x)$

دو تابع مساوی هستند ✓

$$f(x) = \left[\frac{x^2}{x^2 + 1} \right] \text{ و } g(x) = [x - [x]]^2$$

$$D_f \Rightarrow x^2 + 1 \neq 0 \Rightarrow x^2 \neq -1 \Rightarrow \text{همواره برقرار} \Rightarrow D_f = \mathbb{R}$$

$$D_g = \mathbb{R}$$

$$\text{if } x = 0, 1, 2, \dots \Rightarrow f(x) = g(x)$$

دو تابع مساوی نیستند ✓

$$f(x) = \frac{1}{[2x]}, \quad g(x) = \frac{1}{\{[x]\}}$$

$$D_f \Rightarrow [2x] \neq 0 \Rightarrow 2x \neq [0, 1] \Rightarrow x \neq [0, \frac{1}{2}]$$

$$\Rightarrow D_f = \mathbb{R} - [0, \frac{1}{2}]$$

$$D_g \Rightarrow \{[x]\} \neq 0 \Rightarrow [x] \neq 0 \Rightarrow x \neq [0, 1]$$

$$\Rightarrow D_g = \mathbb{R} - [0, 1] \quad \text{دو تابع مساوی نیستند ✓}$$

$$f(x) = \frac{1}{\sqrt{x - |x|}}, \quad g(x) = \frac{1}{\sqrt{|x| - x}}$$

$$D_f \Rightarrow x - |x| \geq 0 \Rightarrow x \geq |x| \Rightarrow x \geq 0$$

$$\Rightarrow D_f = [0, +\infty)$$

$$D_g \Rightarrow |x| - x \geq 0 \Rightarrow \text{همواره برقرار} \Rightarrow D_g = \mathbb{R}$$

$$\Rightarrow D_g = \mathbb{R}$$

دو تابع مساوی نیستند ✓

$$f(x) = \frac{x^3 - x}{x - 1}$$

$$x \neq 1$$

$$x \neq 1$$

$$g(x) = x^2 + x$$

$$D_f = \mathbb{R}$$

$$D_g = \mathbb{R}$$

دو تابع مساوی نیستند ✓

$$\text{if } x = 1, 2, 3, \dots \Rightarrow f(x) = g(x)$$

$$f(x) - g(x) = x^r - x + 1$$

$$f(x) + g(x) = x^r + x + 1$$

$$f(x) = x^r + 1 \Rightarrow$$

$$f(x) = x^r + 1 \Rightarrow g(x) = x$$

$$f'(x) = (x^r + 1)' = x^{r-1} + 1 + rx^{r-1}$$

$$g'(x) = x^r \Rightarrow f'(x) - g'(x) = x^{r-1} + 1 + rx^{r-1} - x^r = \boxed{x^{r-1} + 1 + rx^{r-1}}$$

$$f \times g = (x - \sqrt{x^r - 1}) \times (x + \sqrt{x^r - 1}) = x^r - x^r - 1 = \boxed{-1}$$

$$f(x) = \frac{ax + r}{x^r - mx + n}$$

$$g(x) = \frac{x - b}{x^r - rx - \omega}$$

$$\Rightarrow rx^r - rx - \omega \neq 0 \Rightarrow$$

$$\cancel{ax + r} = \cancel{ax + r}$$

$$x^r - rx - 1 \neq 0$$

$$(x - \omega)(x + r) \neq 0$$

$$\Rightarrow x \neq \frac{\omega}{r}, x \neq -1$$

$$\Rightarrow x^r - mx + n = 0 \Rightarrow x = 1 \Rightarrow 1 - m + n = 0$$

$$\Rightarrow \cancel{ax + r} = \cancel{ax + r}$$

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$$\cancel{ax + r} = \cancel{ax + r}$$

$$(x - \frac{\omega}{r}) \mid (x + 1) = x^r - 1/\omega x - 1/\omega$$

$$\Rightarrow m = 1/\omega$$

$$n = -1/\omega$$

$$x^r - 1/\omega x - 1/\omega = \frac{1}{r} x^r - rx - \omega$$

$$\Rightarrow ax + r = \frac{1}{r}(x - b)$$

$$\Rightarrow \boxed{a = \frac{1}{r}}, \boxed{b = -r}$$

$$am - bn = \frac{r}{\omega} \cdot (-1) = \boxed{-\frac{r}{\omega}}$$

$$f(x) = \frac{bx + r}{\wedge x + b}$$

$$g(x) = c$$

$$\wedge x + b \neq 0$$

$$\wedge x \neq -b$$

$$x \neq \frac{-b}{\wedge} \Rightarrow \frac{-b}{\wedge} = a$$

$$-b = \wedge a$$

$$b = -\wedge a$$

$$rg = \{ (r, \wedge), (r, 1r), (-1, -\wedge), (a, 0) \} \quad \text{IV}$$

~~$$fg = \{ (r, \wedge), (r, 1r), (-1, -\wedge) \}$$~~

~~$$fg = \{ (r, \wedge), (r, 1r), (-1, -\wedge), (a, 0) \}$$~~

$$f = \{ (-1, r), (r, \varepsilon), (r, -r), (a, \frac{1}{r}) \}$$

$$f + g = \{ (r, \wedge), (r, \varepsilon), (-1, -r) \}$$

$$\frac{rg}{f+g} = \{ (r, 1), (r, r), (-1, \varepsilon) \} \Rightarrow R_f = 1, r, \varepsilon$$

$$\boxed{a = -1}$$

$$\boxed{d = a = -1}$$

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$$1 = ra - rb = -r - rb \Rightarrow -rb = \varepsilon$$

$$\boxed{b = -r}$$

$$a - rb = c$$

$$-1 + r = c$$

$$\boxed{d + c = -1 + c = \varepsilon}$$

$$f(x) = \sqrt{-x^2 + x - 4}$$

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$$-x^2 + x - 4 \geq 0$$

$$-x^2 + x \geq 4$$

$$x(x+1) \geq 0$$

$$f(x) = \frac{2x + a}{x^2 + 2x + b}$$

$$g(x) = \frac{2}{x - c}$$

11.

$$x - c \neq 0$$

$$x \neq c$$

$$Dg = \mathbb{R} \setminus \{c\}$$

$\Rightarrow$ دارای ریشه

$$\Delta = 0$$

$$c = -b$$

$$x^2 + 4x + b \Rightarrow \Delta = b^2 - 4ac$$

$$= 16 - 4b \Rightarrow b = 4$$

$$\Rightarrow x^2 + 4x + a = (x + 2)^2$$

$$\Rightarrow f(x) = \frac{2x + a}{x^2 + 4x + 4}$$

$$(x + 2)^2 g(x) = \frac{2}{x + 2}$$

$$c = -2$$

$$\Rightarrow 2x(x + 2) = 2x + 4 \Rightarrow a = 4$$

$$a + b + c = 12$$