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الف) $y = \frac{\sin x - 2}{2 \cos x + 1} \rightarrow 2 \cos x + 1 \neq 0 \Rightarrow 2 \cos x \neq -1 \Rightarrow \cos x \neq -\frac{1}{2}$
 $\Rightarrow x \neq \frac{2\pi}{3}, \frac{4\pi}{3}, \dots \Rightarrow D_f = \mathbb{R} - \left\{ 2k\pi + \frac{2\pi}{3}, 2k\pi + \frac{4\pi}{3} \right\}$

ب) $y = \frac{\sin x + 3}{\cos x - 1} \rightarrow \cos x - 1 \neq 0 \Rightarrow \cos x \neq 1 \Rightarrow x \neq 0, 2\pi, \dots$
 $\Rightarrow D_f = \mathbb{R} - \{ 2k\pi \}$

الف) $y = \frac{2 \sin x + 1}{\tan x + 1} \rightarrow \begin{cases} \cos x \neq 0 \Rightarrow x \neq \frac{\pi}{2}, \frac{3\pi}{2}, \dots \\ \tan x + 1 \neq 0 \Rightarrow \tan x \neq -1 \Rightarrow x \neq \frac{3\pi}{4}, \frac{7\pi}{4}, \dots \end{cases}$
 $\Rightarrow D_f = \mathbb{R} - \left\{ k\pi + \frac{\pi}{2}, k\pi - \frac{\pi}{4} \right\}$

ب) $y = \frac{\cos x + 1}{\cot x - 1} \rightarrow \begin{cases} \sin x \neq 0 \Rightarrow x \neq 0, \pi, 2\pi, \dots \\ \cot x - 1 \neq 0 \Rightarrow \cot x \neq 1 \Rightarrow x \neq \frac{\pi}{4}, \frac{5\pi}{4}, \dots \end{cases}$
 $\Rightarrow D_f = \mathbb{R} - \left\{ k\pi, k\pi + \frac{\pi}{4} \right\}$

الف) $\sin y = x^2 - 2 \rightarrow \begin{cases} x^2 - 2 \geq -1 \Rightarrow x^2 \geq 1 \Rightarrow x = (-\infty, -1] \cup [1, +\infty) \\ x^2 - 2 \leq 1 \Rightarrow x^2 \leq 3 \Rightarrow x = [-\sqrt{3}, \sqrt{3}] \end{cases}$
 $\Rightarrow D_f = [-\sqrt{3}, -1] \cup [1, \sqrt{3}]$

ب) $y = \arccos(\sqrt{x} - 3) \rightarrow \begin{cases} \sqrt{x} - 3 \geq -1 \Rightarrow \sqrt{x} \geq 2 \Rightarrow x \geq 4 \\ \sqrt{x} - 3 \leq 1 \Rightarrow \sqrt{x} \leq 4 \Rightarrow x \leq 16 \end{cases}$
 $\Rightarrow D_f = [4, 16]$

الف) $\cos y = |x| - 3 \rightarrow \begin{cases} |x| - 3 \geq -1 \Rightarrow |x| \geq 2 \Rightarrow x = (-\infty, -2] \cup [2, +\infty) \\ |x| - 3 \leq 1 \Rightarrow |x| \leq 4 \Rightarrow x = [-4, 4] \end{cases}$
 $\Rightarrow D_f = [-4, -2] \cup [2, 4]$

ب) $y = \arcsin(x^2 + 3x + 1) \rightarrow x^2 + 3x + 1 \geq -1 \Rightarrow x^2 + 3x + 2 \geq 0 \Rightarrow (x+1)(x+2) \geq 0 \Rightarrow x \in (-\infty, -2] \cup [-1, +\infty)$
 $x^2 + 3x + 1 \leq 1 \Rightarrow x^2 + 3x \leq 0 \Rightarrow x(x+3) \leq 0 \Rightarrow x \in [-3, 0]$
 $D_f = [-3, -2] \cup [-1, 0]$

الف) $y = \log_{\frac{1}{3}}(x^2 - 4) \rightarrow x^2 - 4 > 0 \Rightarrow x^2 > 4 \Rightarrow x > \pm 2 \Rightarrow x = (-\infty, -2) \cup (2, +\infty)$
 $\Rightarrow D_f = (-\infty, -2) \cup (2, +\infty)$

ب) $y = \log_{\frac{1}{2}}(2 - |x|) \rightarrow 2 - |x| > 0 \Rightarrow 2 > |x| \Rightarrow x = (-2, 2)$
 $\Rightarrow D_f = (-2, 2)$

$$\text{الف) } y = \log \frac{\Delta - x}{x - 2} \rightarrow \Delta - x > 0 \Rightarrow \Delta > x$$

$$x - 2 < x - 2 \neq 1 \Rightarrow x \neq 2$$

$$x - 2 > 0 \Rightarrow x > 2$$

$$\Rightarrow D_f = (2, \Delta) - \{2\}$$

$$\text{ب) } y = \log x^2 - 1 \rightarrow x^2 - 1 > 0 \Rightarrow x^2 > 1 \Rightarrow x = (-\infty, -1) \cup (1, +\infty)$$

$$x + 2 < x + 2 \neq 1 \Rightarrow x \neq -2$$

$$x + 2 > 0 \Rightarrow x > -2$$

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$$D_f = (1, +\infty) \cup (-2, -1) - \{-2\}$$

$$\text{الف) } y = \log \frac{x^2 - 4x + 3}{x - 2} \rightarrow \frac{(x-1)(x-3)}{x-2} > 0 \rightarrow \frac{1}{- \phi + \phi +} \searrow$$

$$\text{ب) } y = \log \frac{x+3}{x-2} \rightarrow \frac{x+3}{x-2} > 0 \rightarrow \frac{-3}{+ \phi - \phi +} \searrow$$

$$D_f = (1, +\infty) - \{2\}$$

$$x = (-\infty, -3) \cup (2, +\infty)$$

$$x+2 < x+2 \neq 1 \Rightarrow x \neq -3$$

$$x+2 > 0 \Rightarrow x > -2$$

✓

$$D_f = (2, +\infty) \cup (-2, -3) - \{-3\}$$

$$\text{الف) } y = \sqrt{3 - \log x} \rightarrow \begin{cases} x - 2 > 0 \Rightarrow x > 2 \\ 3 - \log x > 0 \Rightarrow 3 > \log x \Rightarrow x < 10 \end{cases}$$

$$\text{ب) } y = \log (2 \log x - 1) \rightarrow \begin{cases} x > 0 \\ 2 \log x - 1 > 0 \Rightarrow 2 \log x > 1 \Rightarrow \log x > \frac{1}{2} \Rightarrow x > \sqrt{10} \end{cases}$$

$$D_f = (\sqrt{10}, +\infty)$$

$$D_f = (2, 10]$$

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$$\text{الف) } y = \frac{3}{4^x + 1} \rightarrow 4^x + 1 \neq 0 \Rightarrow 4^x \neq -1 \rightarrow \text{دائماً برقرار است} \Rightarrow D_f = \mathbb{R}$$

$$\text{ب) } y = \frac{3}{4^x - 1} \rightarrow 4^x - 1 \neq 0 \Rightarrow 4^x \neq 1 \Rightarrow x \neq 0 \Rightarrow D_f = \mathbb{R} - \{0\}$$

$$\text{ج) } y = \frac{3}{4^x - 2} \rightarrow 4^x - 2 \neq 0 \Rightarrow 4^x \neq 2 \Rightarrow x \neq \frac{1}{2} \Rightarrow D_f = \mathbb{R} - \{\frac{1}{2}\}$$

$$\text{د) } y = \frac{3}{4^x - 3} \rightarrow 4^x - 3 \neq 0 \Rightarrow 4^x \neq 3 \Rightarrow x \neq \log_4 3 \Rightarrow D_f = \mathbb{R} - \{\log_4 3\}$$

$$\text{الف) } y = (fn + 1)! \rightarrow fn + 1 \in \mathbb{W} \Rightarrow fn \in \mathbb{W} - 1 \Rightarrow n \in \frac{\mathbb{W} - 1}{f}$$

$$\Rightarrow D_f = \{n | n = \frac{k-1}{f}, k \in \mathbb{W}\}$$

$$\text{ب) } y = \left(\frac{3n-2}{2n-5} \right)! \rightarrow \frac{3n-2}{2n-5} \in \mathbb{W} \Rightarrow 2\mathbb{W}n - 5\mathbb{W} = 3n - 2$$

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$$\Rightarrow 2\mathbb{W}n - 3n = 5\mathbb{W} - 2$$

$$\Rightarrow n(2\mathbb{W} - 3) = 5\mathbb{W} - 2$$

$$\Rightarrow n = \frac{5\mathbb{W} - 2}{2\mathbb{W} - 3} \Rightarrow D_f = \{n | n = \frac{5k-2}{2k-3}, k \in \mathbb{W}\}$$