

الف) $y = \sqrt{4-x} \rightarrow 4-x \geq 0 \Rightarrow x \leq 4$
 $\rightarrow 4-\sqrt{4-x} \geq 0 \Rightarrow 4 \geq \sqrt{4-x} \Rightarrow 16 \geq 4-x \Rightarrow x \geq -12$ } $D_f = [-12, 4]$

ب) $y = \sqrt{3-\sqrt{x-2}} \rightarrow x-2 \geq 0 \Rightarrow x \geq 2$
 $\rightarrow 3-\sqrt{x-2} \geq 0 \Rightarrow 3 \geq \sqrt{x-2} \Rightarrow 9 \geq x-2 \Rightarrow 11 \geq x$ } $D_f = [2, 11]$

الف) $y = \sqrt{4-2x^2} \rightarrow \sqrt{(2-\sqrt{2}x)(2+\sqrt{2}x)} \cdot \frac{-\sqrt{2} \sqrt{2}}{-\phi + \phi -} \Rightarrow D_f = [-\sqrt{2}, +\sqrt{2}]$

ب) $y = \sqrt{3|x|-9} \rightarrow \begin{cases} 3x-9 \geq 0 \Rightarrow 3x \geq 9 \Rightarrow x \geq 3 \\ 3x-9 \geq 0 \Rightarrow -9 \geq 3x \Rightarrow -3 \geq x \end{cases} D_f = (-\infty, -3] \cup [3, +\infty)$

الف) $y = \sqrt[3]{\frac{|x|+1}{|x|-3}} \rightarrow |x|-3 \neq 0 \rightarrow \begin{cases} x-3 \neq 0 \Rightarrow x \neq 3 \\ -x-3 \neq 0 \Rightarrow x \neq -3 \end{cases} D_f = \mathbb{R} - \{\pm 3\}$

ب) $y = \sqrt[3]{\frac{\sqrt{x+1}}{\sqrt{x}-3}} \rightarrow \begin{cases} x \geq 0 \\ \sqrt{x}-3 \neq 0 \Rightarrow \sqrt{x} \neq 3 \Rightarrow x \neq 9 \end{cases} D_f = [0, +\infty) - \{9\}$

الف) $y = \frac{\sqrt{3-|x|}}{|x|+2} \rightarrow \begin{cases} 3-|x| \geq 0 \Rightarrow 3 \geq |x| \\ |x|+2 \neq 0 \Rightarrow |x| \neq -2 \end{cases} D_f = [-3, 3]$

ب) $y = \frac{\sqrt{4-x^2}}{|x|-1} \rightarrow \begin{cases} 4-x^2 \geq 0 \Rightarrow (2-x)(2+x) \geq 0 \\ |x|-1 \neq 0 \Rightarrow \begin{cases} x-1 \neq 0 \Rightarrow x \neq 1 \\ -x-1 \neq 0 \Rightarrow x \neq -1 \end{cases} \end{cases} D_f = [-2, 2] - \{\pm 1\}$

الف) $y = \frac{x+1}{\sqrt{x+|x|}} \rightarrow x+|x| \geq 0 \rightarrow \begin{cases} x-x \geq 0 \Rightarrow * \\ x+x \geq 0 \Rightarrow 2x \geq 0 \Rightarrow x \geq 0 \end{cases} D_f = (0, +\infty)$

ب) $y = \frac{1}{\sqrt{x|x|}} \rightarrow x|x| \geq 0 \rightarrow \begin{cases} x^2 \geq 0 \Rightarrow x \geq 0 \\ -x^2 \geq 0 \Rightarrow * \end{cases} D_f = (0, +\infty)$

$$\text{الف) } y = \sqrt{2-x} \Rightarrow 2-x \geq 0 \Rightarrow x \leq 2 \Rightarrow D_f = (-\infty, 2]$$

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$$\text{ب) } y = \frac{1}{\sqrt{2-x}} \Rightarrow 2-x > 0 \Rightarrow x < 2 \Rightarrow D_f = (-\infty, 2)$$

$$\text{الف) } y = \frac{1}{x|x|} \rightarrow x|x| \neq 0 \left\{ \begin{array}{l} -x^2 \neq 0 \Rightarrow x \neq 0 \\ x^2 \neq 0 \Rightarrow x \neq 0 \end{array} \right\} D_f = \mathbb{R} - \{0\}$$

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$$\text{ب) } y = \frac{1}{\sqrt{-x|x|}} \rightarrow -x|x| > 0 \rightarrow x|x| < 0 \Rightarrow D_f = (-\infty, 0)$$

$$\begin{aligned} \text{الف) } y &= \sqrt{\left[x - \frac{1}{x}\right] + \left[x + \frac{1}{x}\right]} \rightarrow \left[x - \frac{1}{x}\right] + \left[x + \frac{1}{x}\right] \geq 0 \Rightarrow \left[x + \frac{1}{x} - 1\right] + \left[x + \frac{1}{x}\right] \geq 0 \\ &\Rightarrow \left[x + \frac{1}{x}\right] - 1 + \left[x + \frac{1}{x}\right] \geq 0 \Rightarrow 2\left[x + \frac{1}{x}\right] \geq 1 \Rightarrow \left[x + \frac{1}{x}\right] \geq \frac{1}{2} \Rightarrow D_f = \left[\frac{1}{2}, +\infty\right) \\ \text{ب) } \sqrt{\left[x - \frac{1}{x}\right] + \left[x + \frac{1}{x}\right]} &\Rightarrow \left[x - \frac{1}{x}\right] + \left[-\left(x - \frac{1}{x}\right)\right] \geq 0 \xrightarrow{\text{if}} \begin{cases} a \in \mathbb{Z} & [a] + [-a] = 0 \\ a \notin \mathbb{Z} & [a] + [-a] = -1 \end{cases} \Rightarrow x - \frac{1}{x} \in \mathbb{Z} \\ &\Rightarrow D_f = \left\{x \mid x = k + \frac{1}{k}, k \in \mathbb{Z}\right\} \end{aligned}$$

$$\begin{aligned} \text{الف) } y &= \frac{1}{\sqrt{\sin^2 x - 1}} \rightarrow \sin^2 x - 1 \neq 0 \Rightarrow \sin^2 x \neq 1 \Rightarrow \sin x \neq \pm 1 \Rightarrow \sin x \neq \pm \frac{\sqrt{1}}{\sqrt{1}} \Rightarrow \sin x \neq \pm \frac{\sqrt{1}}{\sqrt{1}} \\ D_f &= \mathbb{R} - \left\{\frac{k\pi}{2} + \frac{\pi}{2}\right\} \end{aligned}$$

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$$\text{ب) } y = \frac{\cot x + 1}{\tan x + 1} \rightarrow \tan x + 1 \neq 0 \Rightarrow \tan x \neq -1 \Rightarrow D_f = \mathbb{R} - \left\{k\pi + \frac{3\pi}{4}\right\}$$

$$\text{الف) } y = \sqrt{2\sin x - 1} \quad 2\sin x - 1 \geq 0 \Rightarrow 2\sin x \geq 1 \Rightarrow \sin x \geq \frac{1}{2} \Rightarrow D_f = \left[2k\pi + \frac{\pi}{6}, 2k\pi + \frac{5\pi}{6}\right]$$

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$$\text{ب) } y = \sqrt{1 - 2\cos x} \rightarrow 1 - 2\cos x \geq 0 \Rightarrow 1 \geq 2\cos x \Rightarrow \frac{1}{2} \geq \cos x \Rightarrow D_f = \left[2k\pi + \frac{\pi}{3}, 2k\pi + \frac{5\pi}{3}\right]$$