

الف) $y = \sqrt{4 - \sqrt{2-x}} = 2-x \geq 0 \rightarrow x \leq 2$
 $4 - \sqrt{2-x} \geq 0 \rightarrow \sqrt{2-x} \leq 4$
 $2-x \leq 16 \rightarrow -14 \leq x$

$$\left\{ \begin{array}{l} -14 \leq x \leq 2 \\ Df = [-14, 2] \end{array} \right.$$

ب) $y = \sqrt{3 - \sqrt{x-2}} = x-1 \geq 0 \rightarrow x \geq 1$
 $3 - \sqrt{x-2} \geq 0 \rightarrow 3 \geq \sqrt{x-2} \rightarrow 9 \geq x-2 \rightarrow 11 \geq x$

$$\left\{ \begin{array}{l} 1 \leq x \leq 11 \\ Df = [1, 11] \end{array} \right.$$

الف) $y = \sqrt{4 - 2x^2} = 4 - 2x^2 \geq 0 \rightarrow 4 \geq 2x^2 \rightarrow 2 \geq x^2 \rightarrow 1 \geq x \rightarrow Df = (-\infty, 1]$

ب) $y = \sqrt{2|x| - 1} = 2|x| - 1 \geq 0 \rightarrow 2 \leq |x| \rightarrow \left. \begin{array}{l} x \geq 2 \\ x \leq -2 \end{array} \right\} Df = (-\infty, -2] \cup [2, +\infty)$

الف) $y = \sqrt{\frac{|x|+1}{|x|-3}}$ $|x| \neq 3$ $x \neq \pm 3$ $Df = \mathbb{R} - \{\pm 3\}$

ب) $y = \sqrt{\frac{\sqrt{x}+1}{\sqrt{x}-3}}$ $\left\{ \begin{array}{l} \sqrt{x} \geq 0 \rightarrow x \geq 0 \\ \sqrt{x}-3 \neq 0 \rightarrow \sqrt{x} \neq 3 \rightarrow x \neq 9 \end{array} \right. Df = \{x \geq 0, x \neq 9\}$

الف) $y = \frac{\sqrt{3-|x|}}{|x|+2}$ $\left\{ \begin{array}{l} 3-|x| \geq 0 \rightarrow |x| \leq 3 \rightarrow -3 \leq x \leq 3 \\ |x|+2 \neq 0 \rightarrow |x| \neq -2 \end{array} \right. Df = [-3, 3]$

ب) $y = \frac{\sqrt{4-x^2}}{|x|-1}$ $\left\{ \begin{array}{l} 4-x^2 \geq 0 \rightarrow x^2 \leq 4 \rightarrow x \leq 2 \text{ (1)} \\ |x|-1 \neq 0 \rightarrow |x| \neq \pm 1 \rightarrow \mathbb{R} - \{\pm 1\} \text{ (2)} \end{array} \right.$

$(1) \cap (2) \rightarrow Df = \{x \leq 2, x \neq \pm 1\} = (-\infty, 2], \{x \neq \pm 1\}$

الف) $y = \frac{x+1}{\sqrt{x+|x|}} = x+|x| > 0 \rightarrow |x| > -x \rightarrow Df = \mathbb{R}^+$

ب) $y = \frac{1}{\sqrt{x|x|}} = x|x| > 0 \rightarrow Df = \mathbb{R}^+$

$$\text{الف) } y = \sqrt{2 - [x]} \quad 2 - [x] \geq 0 \quad y \geq [x] \quad y > x \quad Df = (-\infty, 2)$$

8

$$\text{ب) } y = \frac{1}{\sqrt{2 - [x]}} \quad \sqrt{2 - [x]} > 0 \quad 2 - [x] > 0 \quad y > [x] \quad y > x \quad Df = (-\infty, 2)$$

$$\text{الف) } y = \frac{1}{x[x]} \quad x[x] \neq 0 \quad 0 \leq x < 1 \quad Df = [0, 1)$$

9

$$\text{ب) } y = \frac{1}{\sqrt{-x[x]}} \quad -x[x] > 0 \quad \begin{matrix} \text{⊕} \times \\ \text{⊖} \times \\ \text{⊙} \times \end{matrix} \quad Df = \emptyset$$

$$\begin{aligned} \text{الف) } y &= \sqrt{\left[x - \frac{1}{y}\right] + \left[x + \frac{1}{y}\right]} = \left[x - \frac{1}{y}\right] + \left[x + \frac{1}{y}\right] \geq 0 \rightarrow \left[x + \frac{1}{y} - 1\right] + \left[x + \frac{1}{y}\right] \\ &\rightarrow \left[x + \frac{1}{y}\right] - 1 + \left[x + \frac{1}{y}\right] \geq 0 \rightarrow 2\left[x + \frac{1}{y}\right] \geq 1 \rightarrow \left[x + \frac{1}{y}\right] \geq \frac{1}{2} \rightarrow x + \frac{1}{y} \geq 1 \\ &\rightarrow x \geq \frac{1}{y} \quad Df = \left[\frac{1}{y}, +\infty\right) \end{aligned}$$

10

$$\begin{aligned} \text{ب) } \sqrt{\left[x - \frac{1}{y}\right] + \left[-x + \frac{1}{y}\right]} &= \begin{cases} [x] + [-x] = 0 \rightarrow x \in \mathbb{Z} \\ [x] + [-x] = -1 \rightarrow x \notin \mathbb{Z} \end{cases} \rightarrow x - \frac{1}{y} \in \mathbb{Z} \\ Df &= \left\{x / x = n + \frac{1}{y}, n \in \mathbb{Z}\right\} \end{aligned}$$

$$\begin{aligned} \text{الف) } y &= \frac{1}{y \sin^2 x - 1} = y \sin^2 x - 1 \rightarrow y \sin^2 x \neq 1 \rightarrow \sin^2 x \neq \frac{1}{y} \\ \sin x &\neq \sqrt{\frac{1}{y}} \quad \sin x \neq \frac{\sqrt{y}}{y} \quad Df = \mathbb{R} - \left\{yK\pi + \frac{\pi}{2}\right\}, \left\{yK\pi + \frac{3\pi}{2}\right\} \end{aligned}$$

11

$$\text{ب) } y = \frac{\cot x + 1}{\tan x + 1} \rightarrow Df = \mathbb{R} - \left\{yK\pi, yK\pi + \frac{\pi}{y}, yK\pi + \frac{3\pi}{2}, yK\pi + \pi, yK\pi + \frac{5\pi}{2}, yK\pi + \frac{7\pi}{2}\right\}$$

$$\text{الف) } y = \sqrt{y \sin x - 1} = y \sin x - 1 \geq 0 \quad \sin x \geq \frac{1}{y} \quad Df = \left[yK\pi + \frac{\pi}{y}\right] \cup \left[yK\pi + \frac{3\pi}{2}\right]$$

$$\begin{aligned} \text{ب) } y &= \sqrt{1 - y \cos x} = y \cos x \leq 1 \quad \cos x \leq \frac{1}{y} \quad Df = \left(yK\pi - \frac{\pi}{y} \mid yK\pi - \frac{\pi}{y}\right] \cup \\ &\quad \left[yK\pi + \frac{\pi}{y} \mid yK\pi + \frac{\pi}{y}\right) \end{aligned}$$

12