

الف) تابع است $y^3 - x = 2x^2 \Rightarrow y^3 = 2x^2 + x \Rightarrow y_1^3 = 2x_1^2 + x_1 \rightarrow y_1 = y_2 \rightarrow y = y_1$ $\left\{ \begin{array}{l} y_1^3 = 2x_1^2 + x_1 \\ y_2^3 = 2x_2^2 + x_2 \end{array} \right. \xrightarrow{\text{تعریف ریاضی}}$

ب) $|y| = -x^2 + x + 3 \xrightarrow{\text{مثال نقض}} x = 1 \quad y = \pm 3 \quad |1| \neq 3 \quad \times$ تابع نیست

ج) $\sqrt{x-4} + |y-2| = 0 \Rightarrow \sqrt{x-4} = 0, |y-2| = 0 \Rightarrow \begin{array}{c} 14 \\ 2 \\ 4 \end{array} \quad \checkmark$ تابع است $x = 4, y = 2$

د) $x = \cos y \xrightarrow{\text{مثال نقض}} x = 0 \quad y = 2k\pi \pm \frac{\pi}{2} \quad |2k\pi + \frac{\pi}{2}| \neq 0 \quad |2k\pi - \frac{\pi}{2}| \neq 0 \quad \times$ تابع نیست

الف) $y = \frac{x+4}{x^2-4x} = \frac{x+4}{x(x-4)} \xrightarrow{\text{ریشه ها}} \begin{array}{c} -4 \\ 0 \\ 4 \end{array} \quad D_f = \mathbb{R} - \{4, 0\}$

ب) $y = \frac{x+4}{x^2-7x+14} \xrightarrow{(-1)^2 - 4(1)(14) = -15} \Delta < 0 \Rightarrow$ ریشه ندارد $\Rightarrow D_f = \mathbb{R}$

ج) $y = \frac{x+4}{x^2+x+4} \xrightarrow{\Delta = (1)^2 - 4 = -15 < 0} \Rightarrow$ ریشه ندارد $\Rightarrow D_f = \mathbb{R}$

د) $y = \frac{x+4}{x^4-4x^2} = \frac{x+4}{x^2(x-2)(x+2)} \xrightarrow{\text{ریشه ها: } -4, 2, -2} \begin{array}{c} -4 \\ -2 \\ 2 \end{array} \quad D_f = \mathbb{R} - \{2, -2, 0\}$

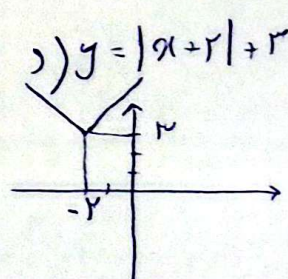
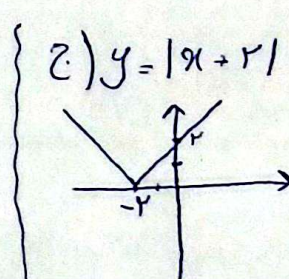
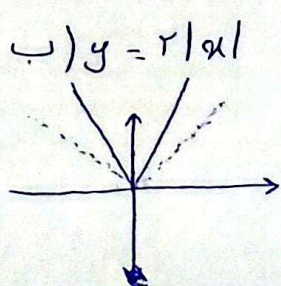
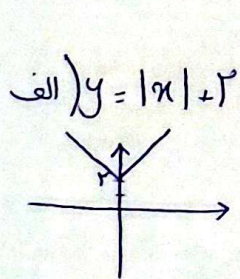
الف) $y = \sqrt{4-x} + \sqrt{x-2} \quad \begin{array}{l} 4-x \geq 0 \\ 4 \geq x \end{array}, \quad \begin{array}{l} x-2 \geq 0 \\ x \geq 2 \end{array} \quad D_f = [2, 4]$

ب) $y = \frac{x+1}{x^2+1} \Rightarrow x^2+1 \neq 0 \Rightarrow x^2 \neq -1 \Rightarrow x^2 \geq 0 \Rightarrow D_f = \mathbb{R}$

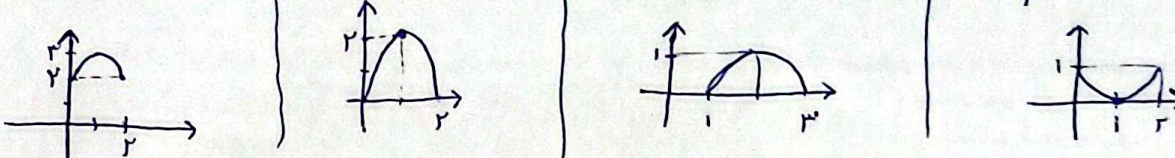
ج) $y = \frac{\sqrt{x-2}}{x-4} \quad \begin{array}{l} x-4 \neq 0 \\ x \neq 4 \end{array}, \quad \begin{array}{l} x-2 \geq 0 \\ x \geq 2 \end{array} \quad D_f = [2, 4) \cup (4, +\infty)$

د) $y = \frac{2x+3}{|x|+x^2} \quad \left\{ \begin{array}{l} |x| \neq 0 \\ x^2 \neq 0 \end{array} \right\} \Rightarrow x \neq 0 \Rightarrow D_f = \mathbb{R} - \{0\}$

$y = |x| \quad \nleftrightarrow$



الف) $f(x) = 2$ ب) $2f(x)$ ج) $f(x-1)$ د) $-f(x)$



الف) $y = (x-2)(x-3)(x-4)$ $\begin{array}{c|ccc} & 2 & 3 & 4 \\ \hline & - & + & - \end{array}$

ب) $y = (x-2)(x-3)^2(x-4)$ $\begin{array}{c|ccc} & 2 & 3^* & 4 \\ \hline & + & - & - \end{array}$

الف) $y = \frac{(x-2)(x-4)}{(x-3)}$ $\begin{array}{c|ccc} & 2 & 3^* & 4 \\ \hline & - & + & - \end{array}$

ب) $y = \frac{(x-2)(x-4)}{(x-3)^2}$ $\begin{array}{c|ccc} & 2 & 3^* & 4 \\ \hline & + & - & - \end{array}$

الف) $\frac{x^3 - 3x + 2}{x - 2} = 0$ $\begin{array}{c|ccc} & -2 & 1^* & 2 \\ \hline & + & - & - \end{array}$

$\frac{(x-1)(x^2+x-2)}{x-2} = \frac{(x-1)(x-1)(x+2)}{(x-2)}$

ب) $\frac{x^3 - 3x + 2}{x^2 - 4x + 2} = \frac{(x-1)(x-2)}{(x-1)(x-3)}$ $\begin{array}{c|ccc} & 1^* & 2 & 3 \\ \hline & + & - & - \end{array}$

الف) $\frac{x^2 - x}{x^2 + x + 2} = \frac{x(x-1)(x+1)}{(x^2 + x + 2)}$ $\begin{array}{c|ccc} & -1 & 0^* & 1 \\ \hline & + & - & - \end{array}$

ب) $\frac{x^2 + 3x + 2}{x^2 + 2x + 2}$ $\Delta = 9 - 4 = 5 > 0$ $\Delta = 4 - 4 = 0$

ج) $\frac{x^2 + 2x + 2}{x^2 + 2x + 2}$ $\Delta = 4 - 4 = 0$

الف) $y = \sqrt{\frac{x^3 - 1}{x^3 - x}} = \sqrt{\frac{(x-1)(x^2+x+1)}{x(x-1)(x+1)}}$ $\begin{array}{c|ccc} & -1 & 0 & 1^* \\ \hline & + & - & + \end{array}$

$D_f = (-\infty, -1) \cup (0, 1) \cup (1, +\infty)$

ب) $y = \sqrt{\frac{x^2 - 14}{x^2 - 5x + 4}} = \sqrt{\frac{(x+4)(x-4)}{(x-1)(x-4)}}$

$\Rightarrow$ $\begin{array}{c|ccc} & -4 & 1 & 4^* \\ \hline & + & - & + \end{array}$ $D_f = (-\infty, -4] \cup (1, 4) \cup (4, +\infty)$