

الف) $y = 2x^2 - 2x$

$x(2x-2) \rightarrow x_1=0, x_2=1$ $c=0$

پارابول سهمی بر دهم A تکلیف ۲۵
از ناحیه سوم نمی گذرد ✓ (۱)

ب) $y = -x^2 + 2x$

$x(-x+2) \rightarrow x_1=0, x_2=2$ $c=0$

از ناحیه دوم نمی گذرد ✓ (۲)

الف) $y = 2x^2 - 5x + 2$

$2x^2 - 5x + 2 = 0 \Rightarrow x_1 = \frac{1}{2}, x_2 = 2$ $a > 0$

از نواحی اول، دوم و چهارم نمی گذرد ✓ (۳)

ب) $-2x^2 + 4x - 1 = 0$

$\Delta = 16 - 8 = 8$
 $x_1 = \frac{2+\sqrt{2}}{2}, x_2 = \frac{2-\sqrt{2}}{2}$

از نواحی اول، سوم و چهارم نمی گذرد ✓ (۴)

$y = x^2 - x - 3$

$-\frac{b}{a} = \alpha + \beta = 1$ $\frac{c}{a} = \alpha\beta = -3$ $|\alpha - \beta| = \frac{\sqrt{\Delta}}{|a|} = \sqrt{13}$

الف) $\frac{\alpha + \beta}{\alpha - \beta} = \frac{1}{\sqrt{13}} = \frac{\sqrt{13}}{13}$ ✓

ب) $\alpha^2 + \beta^2 = 9$ $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 1 - 2(-3) = 7$ ✓ $(1)^2 - 2(-3) = 7$

(۱) (۳)

ج) $\alpha^2 + \beta^2 = (\alpha + \beta)(\alpha + \beta - \alpha\beta) = 1 \times (1 - (-3)) = 4$
 $S^2 - 4SP = 1 - 4(1)(-3) = 13$

د) $\alpha^2 - \beta^2 = (\alpha - \beta)(\alpha + \beta + \alpha\beta) = \sqrt{13}(\alpha - \beta) = \sqrt{13} \times \frac{1}{\sqrt{13}} = 1$

$y = (x-2)(x^2 - ax + a)$

$\Delta = 0 \Rightarrow a^2 - 4a < 0 \Rightarrow a(a-4) < 0$

$a \in (0, 4]$ ✓ (۲) (۴)

$2x^2 - 12x - a = 0 \rightarrow a = 2x^2 - 12x \Rightarrow x^2 - 6x = \frac{a}{2} \Rightarrow x^2 - 6x + 9 = \frac{a}{2} + 9$

$2\alpha^2 + \beta^2 - 4\alpha = 7 \Rightarrow \frac{2\alpha^2}{2} - 4\alpha + \alpha^2 + \beta^2 = 7 \Rightarrow 14 + a = 7 \Rightarrow a = -7$

$-\frac{9}{2} = -3$ ✓ (۲) (۳)

$2x^2 - 12x + 9 = 0 \Rightarrow x^2 - 6x + 4.5 = 0 \Rightarrow (x-1)(x-3) = 0$

$A(2a+3, a-2)$

$a-2 \geq 1$ $a \geq 3$

$B(4-a, a-2)$

$4-a \geq 1$ $a \leq 3$

$\left. \begin{matrix} a \geq 3 \\ a \leq 3 \end{matrix} \right\} a = 3$ $A(9, 1)$ $B(1, 1)$ $x_c = \frac{9+1}{2} = 5$

(۲) (۴)

$S(b, b-2) \Rightarrow S(5, 3)$

$y = a(x-5)^2 + 3 \xrightarrow{(1,1)} 1 = a(1-5)^2 + 3 \Rightarrow a = -\frac{1}{4}$

$y = -\frac{1}{4}x^2 - \frac{5}{4}x + \frac{19}{4}$ $c = -\frac{1}{4}$ ✓

$\frac{1}{4} = 0.25$

$ax^2 - ax - b = 0 \xrightarrow{\div a} x^2 - x - \frac{b}{a} = 0 \Rightarrow \alpha + \beta = 1, \alpha\beta = -\frac{b}{a}$

$4\alpha^2 + 2\alpha - 2\beta = 14 \Rightarrow 2\alpha^2 + \alpha - \beta = 7 \Rightarrow \beta = 2\alpha^2 + \alpha - 7$
 $\beta + \alpha = 1 \Rightarrow 2\alpha^2 + 2\alpha - 6 = 1 \Rightarrow 2\alpha^2 + 2\alpha - 7 = 0$
 $\Delta = 4 + 56 = 60$
 $\alpha = \frac{-2 \pm \sqrt{60}}{4} = \frac{-1 \pm \sqrt{15}}{2}$
 $\beta = 1 - \alpha = \frac{1 \mp \sqrt{15}}{2}$
 $\alpha\beta = -\frac{b}{a} = \frac{1 - 15}{4} = -\frac{14}{4} = -\frac{7}{2}$
 $\Rightarrow 14a = 7a + 4b \Rightarrow 7a = 4b$
 $\Rightarrow a = \frac{4}{7}b$

(۲) (۳)

$|\alpha - \beta| = \frac{\sqrt{\Delta}}{|a|} = \frac{\sqrt{1 - \frac{1}{a}}}{1} = \frac{1}{\sqrt{a}} = \frac{\sqrt{a}}{a}$ ✓

$(-a, \beta), (1, \beta)$

$x_c = \frac{-a+1}{2} = -2$

$y = a(x+2)^2 - \frac{1}{4} = ax^2 + 4ax + 4a - \frac{1}{4}$
 $\frac{x=0}{y=\frac{1}{4}} \Rightarrow 4a - \frac{1}{4} = \frac{1}{4} \Rightarrow 4a = \frac{1}{2} \Rightarrow a = \frac{1}{8}$

(۲) (۳)

$\xrightarrow{(1, \beta)} \beta = \frac{1}{8}(1+2)^2 - \frac{1}{8} = \frac{1}{8} \times 9 - \frac{1}{8} = \frac{8}{8} = 1$ ✓

$x^2 + 4x + a = 0$ $x = \frac{-4 \pm \sqrt{16-4a}}{2} \Rightarrow \alpha = -2 + \sqrt{4-a} \Rightarrow \alpha^2 = 11 - a - 4\sqrt{4-a}$
 $\Rightarrow \beta = -2 - \sqrt{4-a} \Rightarrow \beta^2 = 11 - a + 4\sqrt{4-a}$

(۲) (۴)

$2\alpha^2 + 2\beta^2 = 9 - 2a - 4\sqrt{4-a} + 11 - a + 4\sqrt{4-a} = 20 - 2a$
 $\Rightarrow 20 - 2a = 9 - 2a \Rightarrow 11 = 9$ (Contradiction)

$44x^2 - (m+1)x + 1 = 0$ $\alpha\beta = \frac{1}{44}$ $\frac{1}{\sqrt{a}} + \frac{1}{\sqrt{b}} = \frac{\sqrt{b} + \sqrt{a}}{\sqrt{ab}} < 2 \Rightarrow \sqrt{b} + \sqrt{a} = 2\sqrt{ab} \Rightarrow \beta + \alpha + 2\sqrt{\alpha\beta} = 2\alpha\beta$

(۲) (۱۰)

$\Rightarrow \beta + \alpha + \frac{1}{4} = \frac{1}{44} \Rightarrow \alpha + \beta = \frac{1}{44} - \frac{1}{4} = \frac{1-m}{44} \Rightarrow \frac{m+1}{44} = \frac{1-m}{44} \Rightarrow m = -1$

$m^2x^2 + 2mx + 2 = 0 \Rightarrow -x^2 + 2x + 2 = 0 \Rightarrow \alpha, \beta = \frac{2 \pm \sqrt{4+8}}{-2} = -2$ ✓