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نام و نام خانوادگی ملیکا سبقت اللهی پاسخنامه تشریحی تکلیف شماره ۲۳ کلاس نهم سه شنبه ها

۱ (۲)

$x_{\text{راس}} = -\frac{b}{2a} = -1 \Rightarrow b = 2a$

$a(-1)^2 + b(-1) + c = 9 \Rightarrow a - b + c = 9 \Rightarrow a - 2a + c = 9 \Rightarrow c = 9 + a$

$a(3)^2 + b(3) + c \Rightarrow 9a + 3b + c = 1 \Rightarrow 9a + 6a + (9+a)$

$\Rightarrow 16a + 9 = 1 \Rightarrow 16a = -8 \Rightarrow a = -\frac{1}{2} \,,\, b = 2a \Rightarrow b = -1$

$c = 9 + a = 9 - \frac{1}{2} = \frac{17}{2}$

$y = \boxed{-\frac{1}{2}x^2 - x + \frac{17}{2}}$ یا $\boxed{y = -\frac{1}{2}(x+1)^2 + 9}$

۲ (۲)

$2x^2 + mx + m + 6 = 0 \Rightarrow \Delta > 0 \Rightarrow m^2 - 4(2)(m+6) \Rightarrow m^2 - 8m - 48 = 0$

$m \begin{cases} \to 12 \\ \to -4 \end{cases} \Rightarrow m < -4$ یا $m > 12$

$S > 0 \Rightarrow \alpha + \beta = -\frac{b}{a} = -\frac{m}{2} > 0 \Rightarrow m < 0$

$P > 0 \Rightarrow \alpha\beta = \frac{c}{a} = \frac{m+6}{2} > 0 \Rightarrow m + 6 > 0 \Rightarrow m > -6$

اشتراک گرفتم $\Rightarrow \boxed{-6 < m < -4}$

۳ (۱)

$3x^2 + (2m - \cancel{3})x + 2 - m = 0 \quad \alpha + \beta = -\frac{2m-3}{3} \quad \alpha\beta = \frac{2-m}{3}$

دقت! راه درست

$\alpha + \beta = \frac{1}{\alpha\beta} \Rightarrow -\frac{2m-3}{3} = \frac{1}{\frac{2-m}{3}} = \frac{3}{2-m} \Rightarrow -\frac{2m-3}{3} = \frac{3}{2-m} \Rightarrow$

$-(2m-3)(2-m) = 9 \Rightarrow 2m^2 - 7m + 6 = 9 \Rightarrow 2m^2 - 7m - 3 = 0$

$m = \frac{7 \pm \sqrt{73}}{4} \longrightarrow m = \frac{7 + \sqrt{73}}{4}$ قابل قبول است

۴ (۲)

$x = x^2 - 4 \Rightarrow x^2 - x - 4 = 0 \quad x_1 + x_2 = 1 \quad x_1x_2 = -4$

$y_1 = x_1^3 + \frac{1}{x_2} \,,\, y_2 = x_2^3 + \frac{1}{x_1} \Rightarrow y_1 + y_2 = x_1^3 + x_2^3 + \frac{1}{x_1} + \frac{1}{x_2}$

$x_1^3 + x_2^3 = (x_1 + x_2)^3 - 3x_1x_2(x_1 + x_2) = 1^3 - 3(-4)(1) = 13 \,,\, \frac{1}{x_1} + \frac{1}{x_2}$

$= \frac{x_1 + x_2}{x_1x_2} = -\frac{1}{4} \Rightarrow y_1 + y_2 = 13 - \frac{1}{4} = \boxed{\frac{51}{4}} \,,\, y_1y_2 = (x_1^3 + \frac{1}{x_2})(x_2^3 + \frac{1}{x_1})$

$= x_1^3x_2^3 + \frac{x_1^3}{x_1} + \frac{x_2^3}{x_2} + \frac{1}{x_1x_2} \Rightarrow (x_1x_2)^3 + x_1^2 + x_2^2 + \frac{1}{x_1x_2} = y_1y_2 = -\frac{221}{4}$

$\boxed{y^2 - \frac{51}{4}y - \frac{221}{4} = 0}$ یا $\boxed{4y^2 - 51y - 221 = 0}$

۵ (۲)

$\left(\sqrt[3]{x^2} + \frac{1}{\sqrt[3]{x^2}} + 1\right)\left(\sqrt[3]{x^2} - 1\right) = 2\sqrt[3]{x} \qquad \sqrt[3]{x} = x$

$\left(\frac{\sqrt[3]{x^4} + 1 + \sqrt[3]{x^2}}{\sqrt[3]{x^2}}\right)\left(\sqrt[3]{x^2} - 1\right) = 2\sqrt[3]{x}$

$x^2 - 1 = 2x \Rightarrow x^2 - 2x - 1 = 0 \qquad x_1 + x_2 = -\frac{b}{a} = \boxed{2}$

۶ (۲)

$3x^2 - ax + 4 = 0 \qquad x_2 = 3x_1 \qquad x_1 + x_2 = \frac{a}{3} \qquad x_1x_2 = \frac{4}{3}$

$x_1 + 3x_1 = 4x_1 = \frac{a}{3} \Rightarrow a = 12x_1$

$x_1 \cdot 3x_1 = 3x_1^2 = \frac{4}{3} \Rightarrow x_1^2 = \frac{4}{9} \Rightarrow x_1 = \pm\frac{2}{3}$

$x_1 = \frac{2}{3} \Rightarrow a = 12 \times \frac{2}{3} = 8$ اختلاف دو مقدار $a = \boxed{16}$

$x_1 = -\frac{2}{3} \Rightarrow a = 12 \times \left(-\frac{2}{3}\right) = -8$

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۷ (۱,۷۵)

$a > 0$ (1) $\Rightarrow$ max باید داشته باشد $\qquad$ (1) $\cap$ (2) $\rightarrow \varnothing$

$-\frac{b}{a} \geq 0 \Rightarrow \frac{-3-2a}{a} \geq 0 \Rightarrow -\frac{3}{2} \leq a < 0$

در حالتی که $a = 0$ باشد تابع تبدیل به $y = -x$ می‌شود، باز هم از ناحیه‌ی سوم عبور می‌کند.

$\boxed{-1.5 \leq a < 0}$ (2) $\qquad \boxed{\text{به ازای دو مقدار صحیح } a}$

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۸ (۱,۲۵)

$y = x^2 + ax - 2 \Rightarrow -\frac{a}{2} \quad \Rightarrow x^2 + 2x - 2 = 1 \Rightarrow x^2 + 2x - 3 = 0$ $\quad x_1 = 1 \quad x_2 = -3$

$y = -x^2 - 2x + b \Rightarrow -\frac{-2}{-2} = -1$

$-x^2 - 2x + b = 1 \Rightarrow -x^2 - 2x + b - 1 = 0 \Rightarrow -x^2 - 2x + (b-1) = 0$

$x^2 + 2x - b + 1 = 0 \rightarrow x_1 + x_2 = -2 \checkmark \quad x_1x_2 = -b + 1$

$a = 2 \checkmark, \ \cancel{b = -2} \Rightarrow ab = 2 \times (-2) = \boxed{-4}$

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۹ (۲)

$x_1 = y_1 + \frac{1}{2} \qquad 2x^2 - ax + b = 0 \qquad x_1 + x_2 = \frac{a}{2}, \ x_1x_2 = \frac{b}{2}$

$2ax^2 + ax - 6 = 0 \qquad y_1 + y_2 = -\frac{a}{2a} = -\frac{1}{2}, \ y_1y_2 = -\frac{6}{2a} = -\frac{3}{a}$

$x_1 + x_2 = (y_1 + \frac{1}{2}) + (y_2 + \frac{1}{2}) = y_1 + y_2 + 1, \ y_1 + y_2 = -\frac{1}{2}, \ x_1 + x_2 = \frac{a}{2}$

$\frac{a}{2} = -\frac{1}{2} + 1 = \frac{1}{2} \Rightarrow \boxed{a = 1} \qquad x_1x_2 = (y_1 + \frac{1}{2})(y_2 + \frac{1}{2}) = y_1y_2 + \frac{1}{2}(y_1 + y_2) + \frac{1}{4}$

$y_1 + y_2 = -\frac{1}{2}, \ y_1y_2 = -\frac{3}{a} = -3 \Rightarrow x_1x_2 = -3 + \frac{1}{2} \times \left(-\frac{1}{2}\right) + \frac{1}{4} = -3 - \frac{1}{4} + \frac{1}{4} = -3$

$x_1x_2 = \frac{b}{2} \Rightarrow \frac{b}{2} = -3 \Rightarrow b = -6 \qquad \left[\frac{ab}{4}\right] = \frac{1 \times (-6)}{4} = -1.5 \xrightarrow{\text{براکت}} \boxed{-2}$

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۱۰ (۲)

$x^2 + 6x + m = 0 \rightarrow \alpha^2 + 6\alpha + m = 0 \Rightarrow m = -(\alpha^2 + 6\alpha)$

$x^2 + 2x - 3m = 0 \rightarrow \alpha^2 + 2\alpha - 3m = 0 \Rightarrow 3m = \alpha^2 + 2\alpha \Rightarrow m = \frac{\alpha^2 + 2\alpha}{3}$

$-(\alpha^2 + 6\alpha) = \frac{\alpha^2 + 2\alpha}{3} \Rightarrow -3(\alpha^2 + 6\alpha) = \alpha^2 + 2\alpha \Rightarrow -4\alpha^2 - 20\alpha = 0 \Rightarrow \alpha(-4\alpha - 20) = 0$

$\Rightarrow \boxed{\alpha = -5} \qquad m = -(\alpha^2 + 6\alpha) = -((-5)^2 + 6(-5)) = 5 \Rightarrow \boxed{m = 5}$

$x^2 + 6x + 5 = 0 \rightarrow x_1 = -1 \ *, \ x_2 = -5 \qquad x^2 + 2x - 15 = 0 \rightarrow x_1 = 3 \ *, \ x_2 = -5$

$3 - (-1) = \boxed{4}$

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$$\alpha+\beta=\frac{۱}{\alpha\beta} \longrightarrow \frac{-(۲m-۱)}{۳}=\frac{۳}{۲-m} \longrightarrow \begin{cases} m=-۱ & \text{غ ق ق } \Delta<۰ \\ m=\frac{۷}{۲} & \checkmark \end{cases}$$