

نام و نام خانوادگی علی حسینیان پاسخنامه تشریحی تکلیف شماره ۲۳ کلاس A-پایه

$$S(h, k) \rightarrow y = a(x-h)^2 + k \Rightarrow y = a(x - (-1))^2 + 9 \Rightarrow y = a(x+1)^2 + 9$$

$$(2, 1) \rightarrow 1 = a(2+1)^2 + 9 \rightarrow 1 = 9a + 9 \Rightarrow a = -\frac{1}{4} \Rightarrow y = -\frac{1}{4}(x+1)^2 + 9 = \boxed{-\frac{1}{4}x^2 - x + \frac{17}{4}} \quad \checkmark$$

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$$1. \Delta > 0 \rightarrow m^2 - 4(2)(m+3) = m^2 - 8m - 24 > 0 \rightarrow (m-12)(m+2) > 0 \Rightarrow \underline{m < -2 \text{ یا } m > 12}$$

$$2. S > 0 \rightarrow S = -\frac{b}{a} = -\frac{m}{2} > 0 \Rightarrow \underline{m < 0}$$

$$3. P > 0 \rightarrow P = \frac{c}{a} = \frac{m+3}{2} > 0 \Rightarrow \underline{m > -3}$$

از شرط ۳ استفاده می‌کنیم

$$\underline{-3 < m < -2} \quad \checkmark$$

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$$P = \frac{2-m}{2}, S = -\frac{2m-1}{2} \quad S = \frac{1}{P} \rightarrow -\frac{2m-1}{2} = \frac{2}{2-m} \quad \text{طرفین را ضرب می‌کنیم} \rightarrow -(2m-1)(2-m) = 2 \Rightarrow (2m-1)(m-2) = 9$$

$$2m^2 - 4m - m + 2 = 9 \Rightarrow 2m^2 - 5m - 7 = 0 \Rightarrow (2m+2)(m-7) = 0 \Rightarrow m = -1 \text{ یا } m = 7$$

$$* \Delta \geq 0 \quad \begin{cases} m = -1 \rightarrow \Delta = (-3)^2 - 4(2)(2) = 9 - 16 < 0 \quad \times \\ m = 7 \rightarrow \Delta > 0 \quad \checkmark \end{cases} \Rightarrow \underline{m = 7, 0} \quad \checkmark$$

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$$x^2 - x - 4 = 0 \Rightarrow x_1 + x_2 = 1 \quad x_1 x_2 = -4 \quad x^2 = x + 4 \Rightarrow x^3 = x(x+4) = x^2 + 4x = x + 4 + 4x = \underline{\underline{5x + 4}}$$

$$y_1 = x_1^3 + \frac{1}{x_1} \rightarrow y_2 = x_2^3 + \frac{1}{x_2} \rightarrow x_1^3 = 5x_1 + 4, x_2^3 = 5x_2 + 4 \Rightarrow y_1 = 5x_1 + 4 + \frac{1}{x_1}$$

$$y_1 + y_2 = 5(x_1 + x_2) + 8 + \left(\frac{1}{x_1} + \frac{1}{x_2}\right), x_1 + x_2 = 1, \frac{1}{x_1} + \frac{1}{x_2} = \frac{x_1 + x_2}{x_1 x_2} = \frac{1}{-4} = -\frac{1}{4} \Rightarrow y_1 + y_2 = 5(1) + 8 - \frac{1}{4} = \frac{21}{4}$$

$$y_1 x_2 y_2 = (x_1^3 + \frac{1}{x_1})(x_2^3 + \frac{1}{x_2}) \rightarrow x_1^3 x_2^3 + \frac{x_1^3}{x_2} + \frac{x_2^3}{x_1} + \frac{1}{x_1 x_2} = (x_1 x_2)^3 + x_1^2 + x_2^2 + \frac{1}{x_1 x_2} \rightarrow x_1^2 + x_2^2 = 9, \frac{1}{x_1 x_2} = -\frac{1}{4} \Rightarrow y_1 x_2 y_2 = -27 + 9 - \frac{1}{4} = -\frac{59}{4} \Rightarrow \underline{\underline{4t^3 - 59t - 21 = 0}} \quad \checkmark$$

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$$t = \sqrt[3]{27} \Rightarrow (t + \frac{1}{t} + 1)(t-1) = t^2 + 1 + t - t - \frac{1}{t} - 1 = t^2 - \frac{1}{t} \quad u = \sqrt[3]{27} \Rightarrow u^3 - \frac{1}{u^3} = 2u$$

$$u^3 - 1 = 2u^3 \quad y = u^3 = x, x^2 - 2x - 1 = 0 \rightarrow S = -\frac{-2}{1} = \underline{\underline{2}} \quad \checkmark$$

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$$\alpha = x_1, \beta = 3x_1 \quad S = 4x_1 = \frac{\alpha}{\frac{1}{4}} \Rightarrow x_1 = \frac{\alpha}{14} \rightarrow \left(\frac{\alpha}{14}\right)^2 = \frac{\beta}{4} \rightarrow \frac{\alpha^2}{196} = \frac{\beta}{4} \Rightarrow \alpha^2 = 49\beta \Rightarrow \alpha = \pm 7\sqrt{\beta}$$

$$P = 3x_1^2 = \frac{\beta}{3} \Rightarrow x_1^2 = \frac{\beta}{9} \rightarrow \left(\frac{\alpha}{14}\right)^2 = \frac{\beta}{9} \rightarrow \frac{\alpha^2}{196} = \frac{\beta}{9} \Rightarrow \alpha^2 = \frac{196}{9}\beta \Rightarrow \alpha = \pm \frac{14}{3}\sqrt{\beta}$$

$$\alpha_2 - \alpha_1 = |14 - (-14)| = 28 \quad \checkmark$$

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$$\alpha \geq 0, y \geq 0, x < 0$$

$$\alpha, \beta \geq 0 \Rightarrow -\frac{x^2 + 2a}{a} \geq 0 \quad \text{متناقض دارا} \Rightarrow \boxed{0 \text{ مقبولا}} \quad \checkmark$$

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$$x_1 = -\frac{a}{r} \quad x_2 = -\frac{-r}{-r} = -1 \rightarrow -\frac{a}{r} = -1 \Rightarrow a = r$$

$$y = x^2 + ax - 2 \rightarrow x^2 + 2x - 2 = 1 \rightarrow x^2 + 2x - 3 = 0 \Rightarrow x = \begin{cases} 1 \\ -3 \end{cases}$$

$$y = -x^2 - 2x + b \rightarrow 1 = -(1)^2 - 2(1) + b \Rightarrow 1 = -3 + b \Rightarrow b = 4 \Rightarrow a \times b = 2 \times 4 = 8 \quad \checkmark$$

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$$S_2 = -\frac{1}{r}, P_2 = -\frac{r}{a} \quad S_1 = S_2 + 1 \Rightarrow \frac{a}{r} = -\frac{1}{r} + 1 = \frac{1}{r} \Rightarrow a = 1 \Rightarrow P_2 = -r$$

$$S_1 = \frac{a}{r}, P_1 = \frac{b}{r}$$

$$(\alpha + 1, 0)(\beta + 1, 0) = P_2 + 1, 0 S_2 + 1, 0, \frac{b}{r} = -3 + 1, 0(-1, 0) + 1, 0 = -3 - 1, 0 + 1, 0 = -3 \Rightarrow b = -9$$

$$\left[\frac{ab}{r}\right] = \left[\frac{1 \times (-9)}{1}\right] = [-1, 0] = -1 \quad \checkmark$$

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$$\frac{y}{x} = \alpha \rightarrow (x^2 + 9x + m) - (x^2 + 2x - 3m) = 0 \quad 7x + 4m = 0 \Rightarrow x = -\frac{4m}{7} \quad \alpha = -\frac{m}{4}$$

$$(-m)^2 + 9(-m) + m = 0 \Rightarrow m^2 - 8m = 0 \Rightarrow m = \begin{cases} 0 \\ 8 \end{cases} \quad m \neq 0$$

$$\text{معادله اول} = x^2 + 9x + 0 = 0 \Rightarrow -1, -9$$

$$\text{معادله دوم} = x^2 + 2x - 10 = 0 \Rightarrow -5, 8$$

$$\text{تفاوت های غیر مشترک} = 8 - 1 = 7 \quad \checkmark$$

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