۱۸٫۵

۱- ۲

$-\frac{b}{2a} = -1 \Rightarrow b = 2a$

$$\begin{cases} a - b + c = 9 \Rightarrow -a + c = 9 \\ 9a + 3b + c = 1 \Rightarrow 15a + c = 1 \end{cases} \Rightarrow 16a = -8 \Rightarrow a = -\frac{1}{2} \Rightarrow b = -1 \Rightarrow c = \frac{17}{2}$$

$$\Rightarrow y = \frac{-x^2}{2} - x + \frac{17}{2} \Rightarrow y = \frac{-x^2 - 2x + 17}{2}$$ ✓

۲- ۲

$2x^2 - mx + m + 6 = 0 \xrightarrow{\text{دو ریشه}} m^2 - 8m - 48 > 0 \Rightarrow$ (−۴، ۱۲)

$S > 0 \rightarrow \frac{-b}{a} > 0 \Rightarrow \frac{-m}{2} > 0 \Rightarrow m < 0$

$P > 0 \rightarrow \frac{c}{a} > 0 \Rightarrow \frac{m+6}{2} > 0 \Rightarrow m > -6$

−۶ −۴ ۰ ۱۲

$\Rightarrow m \in (-6, -4)$ ✓

۳- ۲

$S = \frac{1}{P} \Rightarrow \frac{-2m+1}{4} = \frac{3}{-m+2} \Rightarrow 2m^2 - 5m + 2 = 9 \Rightarrow \underbrace{2m^2 - 5m - 7 = 0}$

-1 و $\frac{-c}{a} = \frac{7}{2}$ ریشه‌ها

$\Rightarrow m = -1 \text{ or } \frac{7}{2}$

$4x^2 - 4x + 3 = 0 \longrightarrow$ ۲ ریشه حقیقی ندارد ✗

$4x^2 + 6x - \frac{3}{2} = 0 \longrightarrow$ ۲ ریشه حقیقی دارد ✓ $\longrightarrow m = \frac{7}{2}$ ✓

۴- ۱٫۷۵

$S = x_1 + x_2 = 1 \ , \ P = x_1 . x_2 = -4$

$$\Rightarrow S_{\text{جدید}} = x_1^3 + x_2^3 + \frac{1}{x_1} + \frac{1}{x_2} = (x_1 + x_2)\left((x_1+x_2)^2 - 3x_1 x_2\right) + \frac{x_1 + x_2}{x_1 x_2} = 1(1^2 + 12) - \frac{1}{4} = \boxed{12.75}$$

$$\Rightarrow P_{\text{جدید}} = \left(x_1^3 + \frac{1}{x_2}\right)\left(x_2^3 + \frac{1}{x_1}\right) = \underbrace{(x_1 x_2)^3 + x_1^2 + x_2^2 + \frac{1}{x_1 x_2}}$$

دقت! −۵۵٫۲۵

$$(x_1 x_2)^3 + (x_1 + x_2)^2 - 2x_1 x_2 + \frac{1}{x_1 x_2} = -64 + 1 + 8 - \frac{1}{4} = \boxed{\cancel{}}$$

۵۵٫۲۵

$$\Rightarrow 0 = x^2 - Sx + P = \boxed{x^2 - 12.75x - \cancel{}}$$ ✓

۵- ۲

$$\left(\sqrt[3]{x^2} + \frac{1}{\sqrt[3]{x^2}} + 1\right)\left(\sqrt[3]{x} - \frac{1}{\sqrt[3]{x}}\right) = \left(\sqrt[3]{x^2} + \frac{1}{\sqrt[3]{x^2}} + 1\right)\left(\frac{\sqrt[3]{x^2} - 1}{\sqrt[3]{x}}\right) = x - \frac{1}{x}$$

$$\Rightarrow \left(\sqrt[3]{x^2} + \frac{1}{\sqrt[3]{x^2}} + 1\right)\left(\sqrt[3]{x^2} - 1\right) = \left(x - \frac{1}{x}\right)\sqrt[3]{x} = 2\sqrt[3]{x} \Rightarrow x - \frac{1}{x} = 2 \Rightarrow x - 2 - \frac{1}{x} = 0$$

$\Rightarrow x^2 - 2x - 1 = 0$

$\Rightarrow S = \frac{-b}{a} = \boxed{2}$ ✓

۱۷٫۵

$P = \frac{4}{3} \Rightarrow 3a^2 = \frac{4}{3} \Rightarrow a^2 = \frac{4}{9} \Rightarrow a = \pm\frac{2}{3}$

$\Rightarrow \left|\frac{2}{3} - \left(-\frac{2}{3}\right)\right| = \boxed{\cancel{\frac{4}{3}}}$

$x_{ext} \longrightarrow$ منفی باشد
$a > 0$

$\left\{ \frac{-4-2a}{2a} > 0 \longrightarrow (-2, 0) \;\; \text{غیر قابل قبول} \;\; \longrightarrow a > 0 \right\} \xrightarrow{\cap} \varnothing$ ✓ ۲

$\frac{+2}{-2} = \frac{-a}{2} \Rightarrow a = 2$

$1 = x^2 + 2x - 2 \Rightarrow x = 1$

$\Rightarrow -1 - 2 + b = 1 \Rightarrow b = 4$

$\Rightarrow ab = 8$ ✓ ۲

$\frac{a}{2} = \frac{-a}{2a} + 1 = \frac{1}{2} \Rightarrow a = 1$

$(\alpha + 0.5)(\beta + 0.5) = \alpha\beta + 0.5(\alpha + \beta) + 0.25 = P + \frac{S}{2} + 0.25 = \frac{b}{2}$

$\Rightarrow -3 + \cancel{\frac{-1}{4}} + \cancel{\frac{1}{4}} = \frac{b}{2} \Rightarrow b = -6$

$\Rightarrow \left[\frac{ab}{4}\right] = \left[\frac{-6}{4}\right] = \boxed{-2}$ ✓ ۲

$\alpha \longrightarrow$ ریشه مشترک و غیر صفر

$\Rightarrow \begin{cases} \alpha^2 + 2\alpha - 3m = 0 \\ 3\alpha^2 + 18\alpha + 3m = 0 \end{cases} \Rightarrow 4\alpha^2 + 20\alpha = 0 \Rightarrow \alpha = 0 \times \text{ or } \alpha = -5$

$\Rightarrow m = 5$

$x^2 + 6x + 5 \begin{cases} -1 \\ -5 \end{cases}$

$x^2 + 2x - 15 = (x+5)(x-3)$
$-5 \qquad +3$

$|3 - (-1)| = \boxed{4}$ ✓ ۲

-۶

$$P = \alpha \underbrace{\cancel{\beta}}_{۳\alpha} = ۳\alpha^۲ = \frac{۴}{۳} \rightarrow \alpha = \mp \frac{۲}{۳}$$

$$S = \alpha + \underbrace{\cancel{\beta}}_{۳\alpha} = ۴\alpha = \frac{a}{۳} \rightarrow \mp \frac{۸}{۳} = \frac{a}{۳} \rightarrow \boxed{a = \mp ۸}$$

اختلاف $= \boxed{۱۶}$