

۲۰

$$\begin{aligned} \frac{-b}{2a} &= -1 \rightarrow b = 2a \\ \frac{-\Delta}{2a} &= 9 \rightarrow \frac{-b^2 + 4ac}{4a} = \frac{-4a^2 + 4ac}{4a} = C - a = 9 \rightarrow C = a + 9 \\ (C, 1) &\rightarrow a(1)^2 + 1(1a) + a + 9 = 1 \rightarrow 14a = -1 \rightarrow a = -0.07 \\ \rightarrow b &= 2(-0.07) = -0.14, C = 9 - 0.07 = 8.93 \\ y &= -x^2 - 2x + 17 \end{aligned}$$

(۲) ۱

$$\begin{aligned} 2x^2 + mx + m + 4 &= 0 \rightarrow 1) \Delta > 0 \rightarrow m^2 - 4m - 4 > 0 \\ \rightarrow (m - 12)(m + 4) &> 0 \rightarrow m = (-\infty, -4) \cup (12, +\infty) \\ 2) \Delta < 0 &\rightarrow \frac{-m}{2} > 0 \rightarrow m < 0 \\ 3) \Delta = 0 &\rightarrow \frac{m + 4}{2} > 0 \rightarrow m > -4 \\ \text{نتیجه} &: m = (-4, -4) \end{aligned}$$

(۲) ۲

$$\begin{aligned} \alpha + \beta = 5, \alpha\beta = p \\ \delta = \frac{1}{p} \rightarrow -\frac{b}{a} = \frac{c}{c} \rightarrow a' = -b' \rightarrow q = -(2m - 1)(2 - m) \\ = -(4m - 2m^2 - 2 + m) = 2m^2 - 5m + 2 = 9 \rightarrow 2m^2 - 5m - 7 = 0 \\ \rightarrow m^2 - 5m - 14 = 0 \rightarrow (m - 7)(m + 2) = 0 \rightarrow m = 7, -2 \end{aligned}$$

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$$\begin{aligned} \alpha + \beta = 5, \alpha\beta = p \rightarrow x^2 - 5x + p = 0 \\ \delta = \left(\beta^2 + \frac{1}{\alpha}\right) + \left(\alpha^2 + \frac{1}{\beta}\right) = \frac{\alpha\beta^3 + 1}{\alpha\beta} + \frac{\alpha^3\beta + 1}{\alpha\beta} = \frac{\alpha\beta^3 + \alpha^3\beta + \alpha + \beta}{\alpha\beta} = \frac{5 + p(\alpha^2 + \beta^2)}{p} \\ = \frac{5 + p(5^2 - 2p)}{p} = \frac{5 + 25p - 2p^2}{p} = \frac{-2p^2 + 25p + 5}{p} \\ p = \frac{\alpha\beta^3 + 1}{\alpha} + \frac{\alpha^3\beta + 1}{\beta} = \frac{(\alpha\beta)^2 + (\alpha\beta)(\alpha^2 + \beta^2) + 1}{\alpha\beta} = \frac{p^2 + p(5^2 - 2p) + 1}{p} = \frac{p^2 + 25p - 2p^2 + 1}{p} = \frac{-p^2 + 25p + 1}{p} \\ \rightarrow 0 = x^2 - 12.75x - 56.25 \rightarrow x = 15.75, -3.5 \end{aligned}$$

(۲) ۴

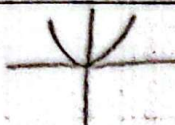
$$\begin{aligned} \left(\sqrt[3]{x^2} + \frac{1}{\sqrt[3]{x^2}} + 1\right)\left(\sqrt[3]{x^2} - 1\right) &= 2\sqrt[3]{x^2} \rightarrow \sqrt[3]{x^2} - \sqrt[3]{x^2} + 1 - \frac{1}{\sqrt[3]{x^2}} + \sqrt[3]{x^2} - 1 \\ &= \sqrt[3]{x^2} - \frac{1}{\sqrt[3]{x^2}} = 2\sqrt[3]{x^2} \times \sqrt[3]{x^2} \rightarrow \sqrt[3]{x^4} - 1 = 2\sqrt[3]{x^2} \\ \rightarrow x^2 - 2x - 1 &= 0 \rightarrow x = 1, -1 \end{aligned}$$

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پیش فرض: $x^2 - \alpha x + \epsilon = 0$ را بر β و ϵ در نظر بگیرید.

$P = \epsilon\beta \times \beta = \epsilon\beta^2 = \frac{\epsilon}{\epsilon} \rightarrow \beta^2 = \frac{\epsilon}{\epsilon} \rightarrow \beta = \pm \frac{\sqrt{\epsilon}}{\sqrt{\epsilon}}$

$S = \epsilon\beta \cdot \beta = \epsilon\beta = \frac{\alpha}{\epsilon} \rightarrow \beta = \frac{\alpha}{\epsilon}$
 $\frac{\alpha}{\epsilon} = \frac{\epsilon}{\epsilon} \rightarrow \alpha = 1$
 $\frac{\alpha}{\epsilon} = -\frac{\epsilon}{\epsilon} \rightarrow \alpha = -1$
 $\Delta\alpha = |1 - (-1)| = \boxed{14}$

حالت ۱:  $\alpha > 0$
 $\Delta = 0 \rightarrow 4\alpha^2 + 12\alpha + 9 = 0 \rightarrow \alpha = -\frac{3}{2}$
 $P = 0, S = 0 \rightarrow -2\alpha - \epsilon = 0 \rightarrow \alpha = -\frac{\epsilon}{2}$

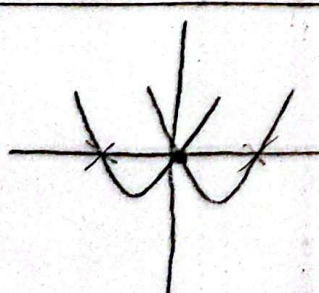
حالت ۲:  $\alpha > 0$
 $\Delta > 0$
 $P = 0$
 $S > 0 \rightarrow -2\alpha - \epsilon > 0 \rightarrow \alpha < -\frac{\epsilon}{2}$

$\frac{-\alpha}{\epsilon} = \frac{\epsilon}{\epsilon} \rightarrow \alpha = \epsilon$
 $x^2 + 2x - 2 = 1 \rightarrow x^2 + 2x - 3 = 0 \rightarrow x = -3, x = 1$
 $-x^2 - 2x + b = 1 \rightarrow -1 - 2 + b = 1 \rightarrow b = 4 \rightarrow ab = 2 \times 4 = \boxed{8}$

نگار: از آنجایی که دو دایره، نقطه تماس دارند، پس دهانه آنها یکسان است.

برای حل معادله $x^2 - \alpha x + b = 0$ به دو دایره از دستگاه معادلات $\begin{cases} x^2 - \alpha x + b = 0 \\ x^2 - \alpha x + \epsilon = 0 \end{cases}$ بپردازیم.

$\frac{\alpha}{\epsilon} = \frac{-\alpha}{\epsilon} + 1 \rightarrow \alpha = -1 + \epsilon \rightarrow \alpha = 1$
 $\alpha\beta = \frac{\epsilon}{\epsilon} = 1, (\alpha + \frac{1}{\epsilon})(\beta + \frac{1}{\epsilon}) = \alpha\beta + \frac{1}{\epsilon} + (\alpha + \beta)\frac{1}{\epsilon} = 1 + \frac{1}{\epsilon} + \frac{-1}{\epsilon} = 1$
 $-3 = \frac{b}{\epsilon} \rightarrow b = -3 \rightarrow \left[\frac{ab}{\epsilon}\right], \left[\frac{1 \times -3}{\epsilon}\right] = \boxed{-3}$

 $S = \alpha + \epsilon = \frac{-b}{a} = \frac{-\epsilon}{1} = -\epsilon$
 $S \cdot \beta + \epsilon = \frac{-b}{a} = \frac{-\epsilon}{1} = -\epsilon$
 $(\alpha + \epsilon) - (\beta + \epsilon) = \alpha - \beta = \boxed{4}$