پاسخ تمرین

۱) الف) $\left(\frac{\sqrt{۲}}{۲}-\frac{\sqrt{۳}}{۲}\right)\left(\frac{\sqrt{۲}}{۲}+\frac{\sqrt{۳}}{۲}\right)=\left(\frac{\sqrt{۲}}{۲}\right)^{۲}-\left(\frac{\sqrt{۳}}{۲}\right)^{۲}=\frac{۲}{۴}-\frac{۳}{۴}=\boxed{-\frac{۱}{۴}}$

ب) $\frac{۱+۲\times\frac{۱}{۲}}{۲\times\frac{۲}{۴}+۲\times\frac{۱}{۴}}=\frac{۲}{۴}=\boxed{\frac{۱}{۲}}$

۲) الف) $AH=\sin ۴۵^\circ\times\sqrt{۶}=\frac{\sqrt{۲}}{۲}\times\sqrt{۶}=\frac{\sqrt{۱۲}}{۲}=\frac{۲\sqrt{۳}}{۲}=\sqrt{۳}$

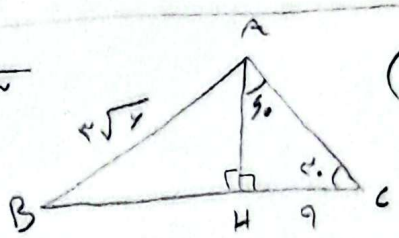
$(۲\sqrt{۳})^{۲}=\sqrt{۳}^{۲}+HC^{۲}=۱۲\quad HC^{۲}=۱۲-۳=۹\quad HC=\boxed{۳}$

ب) $\frac{\sqrt{۲}}{۲}\times AC=۹\Rightarrow AC=\frac{۹}{\frac{\sqrt{۲}}{۲}}=\frac{۱۸}{\sqrt{۲}}=\frac{۱۸\sqrt{۲}}{۲}=۹\sqrt{۲}$

$AH=\frac{۱}{۳}\times۹\sqrt{۲}=۳\sqrt{۲}$

$۲\sqrt{۶}\times\sin B=۳\sqrt{۲}\quad \sin B=\frac{۳\sqrt{۲}}{۲\sqrt{۶}}=\frac{۳\sqrt{۲}}{۲\sqrt{۲}\sqrt{۳}}=\frac{\sqrt{۲}}{۲}$

$\hat{B}=\boxed{۴۵^\circ}$



۳) الف) $\frac{\sqrt{۳}}{۲}\times AD=۵۰\sqrt{۳}\quad AD=۱۰۰$

$\frac{۱}{۲}\times AC=۵۰\sqrt{۳}\quad AC=۱۰۰\sqrt{۳}$

$BD=\frac{۱}{۲}\times۱۰۰=۵۰\quad BC=\frac{\sqrt{۳}}{۲}\times۱۰۰\sqrt{۳}=۱۵۰$

$x=BC-BD=۱۵۰-۵۰=\boxed{۱۰۰}$

ب) $AC\times\frac{۱}{۲}=۲\quad AC=۴\quad BC=\frac{\sqrt{۳}}{۲}\times۴=۲\sqrt{۳}$

$CD^{۲}=۱۲+۱=۱۳\quad CD=\sqrt{۱۳}\quad \sin\alpha=\frac{۱}{\sqrt{۱۳}}=\boxed{\frac{\sqrt{۱۳}}{۱۳}}$

۴) $AC=x\quad CH=\frac{\sqrt{۲}}{۲}x\quad \frac{BH}{۲}=\frac{\sqrt{۲}}{۲}x\times\frac{\sqrt{۳}}{۲}=\frac{\sqrt{۶}}{۴}x$

$BH=\frac{\sqrt{۶}}{۲}x\quad AB=\frac{\sqrt{۶}}{۲}x+\frac{\sqrt{۲}}{۲}x=\frac{\sqrt{۶}+\sqrt{۲}}{۲}x\quad \frac{AC}{AB}=\frac{x}{\frac{\sqrt{۶}+\sqrt{۲}}{۲}x}=\boxed{\frac{۲}{\sqrt{۶}+\sqrt{۲}}}$

۵) $S(\triangle AEB)=\frac{۲\sin B\times۴۲}{۲}\quad S(\triangle BCD)=\frac{۳۶\sin B\times۴}{۲}$

$\frac{S(ABE)}{S(BCD)}=\frac{۴\sin B}{۹\sin B}=\boxed{\frac{۴}{۹}}$

(٤ الف)

$\frac{\sqrt{٣}}{٢} \times ٣ \times \frac{٤}{٢} = \boxed{٣\sqrt{٣}}$

(ب)

$\frac{١}{٢} \times ٣ \times \frac{٤}{٢} \times \frac{\sqrt{٣}}{٢} = \boxed{٣\sqrt{٣}}$

(٧

$٣^٢ + ٤^٢ = ٩ + ١٦ = ٢٥ \qquad \sqrt{٢٥} = \overline{PQ}$

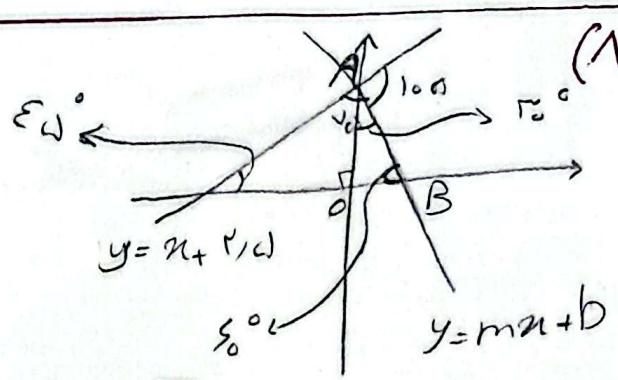
$\cos\theta = \frac{٤}{\sqrt{٢٥}} = \frac{٤\sqrt{٥}}{٢٥} = \boxed{\frac{٢\sqrt{٥}}{٥}}$

(٨

$١٨٠ - ١٠٥ = ٧٥$

$AO = ٢,٥$

$٧٥ - ٤٥ = ٣٠^\circ$



$\frac{\sqrt{٣}}{٢} \times BA = \frac{٥}{٢}$

$BA = ٥\sqrt{٣}$

$BO = \frac{٥\sqrt{٣}}{٢} \Rightarrow b = \frac{٥\sqrt{٣}}{٢}$

$m = \frac{\frac{\sqrt{٣}}{٢}}{\frac{١}{٢}} = \boxed{\sqrt{٣}}$

$mb = \sqrt{٣} \times \frac{٥\sqrt{٣}}{٢} = \boxed{\frac{١٥}{٢}}$

(٩

$f\left(\frac{٥\pi}{٦}\right) = \cos\left(\pi + \frac{٥\pi}{٦}\right) + \sin\left(\frac{\pi}{٢} - \frac{٥\pi}{٦}\right) \cdot \tan\left(\frac{٣\pi}{٢} + \frac{٥\pi}{٦}\right)$

$-\cos\frac{٥\pi}{٦} + \cos\frac{٥\pi}{٦} + \cot\frac{٥\pi}{٦} = \frac{\sqrt{٣}}{٢} + \frac{\sqrt{٣}}{٢} + \sqrt{٣} = \boxed{٢\sqrt{٣}}$

(١٠

$\frac{-\cos x - \sin x}{-\cos x + \sin x}$