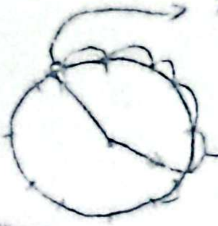
پویان ریاحی

۱۷

۱) الف)



$4 \times 30^\circ = 120^\circ$

$\frac{1}{2} \times 54 = 27^\circ$

$120 + 27 + 6 = \boxed{153^\circ}$ ✓

$360 - 153 = \boxed{207^\circ}$

ب)

$|5.5\, min - 30h| \Rightarrow \frac{11}{2} \times 54 - 30 \times 3 = 297 - 90 = \boxed{207^\circ}$

$360^\circ - 207^\circ = \boxed{153^\circ}$ ✓

همیشه زاویه کوچکتر مدنظره!

(۲)

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۲) الف)

$2 \times 30^\circ = 60^\circ$

$2 \times 6 = 12^\circ$

$18 \times \frac{1}{2} = 9^\circ$

$12 + 9 + 60 = 81$ ✓

(۲)

ب)

$\alpha = \left|\frac{11}{2} min - 30h\right| \Rightarrow \left|\frac{11}{2} \times 18 - 30 \times 6\right| = |99 - 180| = \boxed{81^\circ}$ ✓

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۳) الف)

$\frac{\pi}{6} = 30^\circ \qquad \frac{30}{360} = \frac{1}{12} \qquad S = \pi R^2 \rightarrow 9\pi\, cm^2$

$\frac{1}{12} \times 9\pi = \boxed{\frac{3\pi}{4}\, cm^2} \rightarrow S$ ✓

(۲)

ب)

$P = \pi 2R \rightarrow 6\pi \qquad 6\pi \times \frac{1}{12} = \frac{\pi}{2}$

$\boxed{\frac{\pi}{2} + 6}$ ✓

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۴) الف)

$\frac{5 \times 8 \times \frac{\sqrt{3}}{2}}{2} = \boxed{10\sqrt{3}} \rightarrow S$ ✓

(۲)

$BC = \sqrt{25 + 64 - 80 \times \cos 60} = \sqrt{89 - 40} = \sqrt{49} = 7$ ✓

$P = 7 + 5 + 8 = \boxed{20}$ ✓

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۵)

$\frac{15}{\sin A} = \frac{15\sqrt{2}}{\sin B} = \frac{AB}{\sin C}$

$\frac{15}{\frac{1}{2}} = 30 = \frac{15\sqrt{2}}{\frac{\sqrt{2}}{2}} \Rightarrow B = 45^\circ \qquad C = 180 - 75 = 105^\circ$

$\boxed{\frac{\pi}{4}, \frac{\pi}{6}, \frac{7\pi}{12}}$ ✓

(۲)

۶) $\frac{-\tan\alpha + 2\tan\alpha}{-\tan\alpha - \tan\alpha} = -\frac{2\tan\alpha}{2\tan\alpha} = \boxed{-1}$ ✓ (۲)

۷) $\frac{2\tan\left(\frac{\pi}{2} - \frac{\pi}{12}\right) + \tan\left(\frac{\pi}{2} + \frac{\pi}{12}\right)}{2\tan\left(\pi - \frac{\pi}{12}\right) + \tan\left(\frac{2\pi}{2} + \frac{\pi}{12}\right)} = \frac{2\frac{1}{a} - \frac{1}{a}}{-2a - \frac{1}{a}} = \frac{\frac{1}{a}}{-\frac{2a^2+1}{a}} = \frac{1}{2a^2+1}$ (۲)

$A = \frac{-1}{2a^2+1}$ ✓

۸) $\frac{\sin^2 x + \cos^2 x + 2\sin x\cos x + \sin^2 x + \cos^2 x - 2\sin x\cos x}{\sin^2 x - \cos^2 x} = \frac{2}{\sin^2 x - \cos^2 x} = 3$ (۱)

$\sin^2 x - \cos^2 x \rightarrow \cos 2x = \frac{1}{4}$ دقت!

$\sin^2 x - \cos^2 x = \frac{1}{2} \Rightarrow 1 - 2\cos^2 x = \frac{1}{2} \Rightarrow 2\cos^2 x = \frac{2}{3}$ $\cos^2 x = \frac{1}{3}$ (۱)

$\sin^2 x = \frac{2}{3}$ $\tan^2 x = \frac{\frac{2}{3}}{\frac{1}{3}} = \boxed{2}$

$1 + \tan^2 = \frac{1}{\cos^2 x} \rightarrow \tan^2 = \boxed{5}$

۹) $4\sin^2 x - 4\cos^2 x + 1 = \sin^2 x + \cos^2 x - 1$ (۵)

$2\sin^2 x + 2 = 9\cos^2 x$ $2\sin^2 x + 2 = 9 - 9\sin^2 x \Rightarrow 11\sin^2 x = 7$

$\sin^2 x = \frac{7}{11}$ $\cos^2 x = \frac{4}{11}$ $\tan^2 x = \frac{\frac{7}{11}}{\frac{4}{11}} = \boxed{\frac{7}{4}}$

۱۰) $\cos(11.5) = \sqrt{\frac{1+\cos 30}{2}} = \frac{1+\frac{\sqrt{3}}{2}}{2} = \frac{\sqrt{2+\sqrt{3}}}{2}$ ✓ (۲)

$\sin 97.5 = \sqrt{\frac{1-\cos 15}{2}} = \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}} = \frac{\sqrt{2+\sqrt{3}}}{2}$ ✓

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۱۹) $4\sin^2 x + 8\cos^2 x - 4 = \sin^2 x - 2\cos^2 x + 1$

$2\sin^2 x + 2\cos^2 x + 7\cos^2 x = 5 \rightarrow \cos^2 x = \frac{2}{7}$ (۳)

$1 + \tan^2 = \frac{1}{\cos^2} \rightarrow \tan^2 = \boxed{\frac{5}{2}}$