

$$\frac{\tan \alpha + 1}{\tan \alpha - 1} + \frac{\tan \alpha - 1}{\tan \alpha + 1} = r \Rightarrow \frac{\tan^2 \alpha + 1 + \tan^2 \alpha - 1}{\tan^2 \alpha - 1} = \frac{r(\tan^2 \alpha + 1)}{\tan^2 \alpha - 1} = r$$

$$\Rightarrow (\tan^2 \alpha + 1) \neq (\tan^2 \alpha - 1) \Rightarrow \tan^2 \alpha = r$$

$$\frac{\sin^2 \alpha - r \cos^2 \alpha + \sin^2 \alpha + \cos^2 \alpha}{\sin^2 \alpha + r \cos^2 \alpha - \sin^2 \alpha - \cos^2 \alpha} = \frac{r \sin^2 \alpha - \cos^2 \alpha}{\cos^2 \alpha} = r$$

$$\Rightarrow \frac{r \tan^2 \alpha - 1}{1} = r \Rightarrow r \tan^2 \alpha = r \Rightarrow \tan^2 \alpha = r$$

$$\text{a)} \cos \alpha = \frac{1 + \cos 2\alpha}{2} \Rightarrow \cos^2(\varphi, d) = \frac{1 + \cos 2\varphi}{2} = \frac{1 + \frac{\sqrt{r}}{r}}{2}$$

$$= \frac{r + \sqrt{r}}{2r} = \frac{1}{2} + \frac{\sqrt{r}}{2r} \Rightarrow \cos(\varphi, d) = \frac{\sqrt{r + \sqrt{r}}}{2} \Rightarrow \boxed{\cos(\varphi, d) = \frac{\sqrt{r + \sqrt{r}}}{2}}$$

$$\Rightarrow \sin \alpha = \frac{1 - \cos 2\alpha}{2} \Rightarrow \sin^2(\varphi, d) = \frac{1 - \cos 2\varphi}{2} = \frac{1 - (-\frac{\sqrt{r}}{r})}{2} = \frac{r + \sqrt{r}}{2r}$$

$$\Rightarrow \sin(\varphi, d) = \frac{\sqrt{r + \sqrt{r}}}{2} \Rightarrow \boxed{\sin(\varphi, d) = \frac{\sqrt{r + \sqrt{r}}}{2}}$$