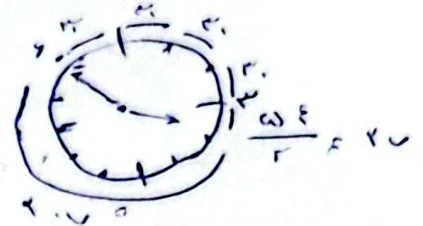


۱۱

۳:۵۴'

الف

$$3-44+27+4=153$$



$$a = |\omega_{\text{سم}} - \omega_{\text{H}}| = |\omega_{\text{سم}}(1 - \frac{1}{12})| = 2.7$$

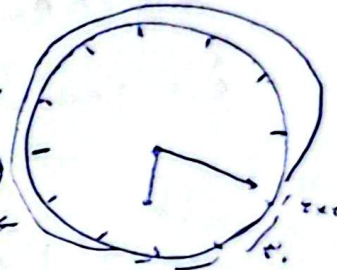
از اونجا که

$$315 - 27 = 288 = 12 \times 24$$

۲۱

۶:۱۸'

الف



$$3 \times 2 + 18 + 9 = 11$$

ب

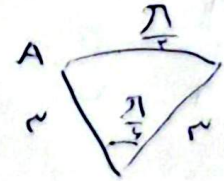
$$a = |\omega_{\text{سم}} - \omega_{\text{H}}| = |\omega_{\text{سم}}(1 - \frac{1}{12})| = 11$$

از اونجا که

۳۱

$$||\text{الف}|| = \frac{a}{r} R' = \frac{\pi}{12} \times r' = \frac{3\pi}{2}$$

$$|AB| = aR = \frac{\pi}{12} \times r = \frac{\pi}{12}$$



$$\Rightarrow \text{محیط} = \frac{\pi}{12} + r + r = 2 + \frac{\pi}{12}$$

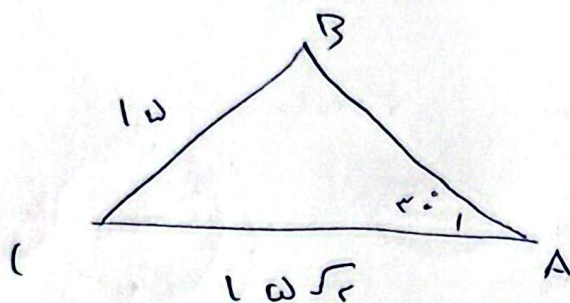


$$S_A = \frac{1}{2} ab \sin \hat{A} = \frac{1}{2} \times 8 \times 8 \times \sin 45^\circ = 16\sqrt{2}$$

$$a = \sqrt{b^2 + c^2 - 2bc \cos \hat{A}} = 8 \sqrt{2(1 + 1 - \frac{1}{2})} = \sqrt{96}$$

$$\Rightarrow \text{محیط} = \sqrt{96} + 8 + 8 = 20$$

ω)



$$\hat{B} + \hat{C} = 100^\circ$$

$$\hat{A} = 180^\circ - \hat{B} - \hat{C} = 180^\circ - 100^\circ = 80^\circ$$

$$\frac{\sin \hat{B}}{b} = \frac{\sin \hat{A}}{a} = \frac{1}{1\omega} = \frac{1}{r} \Rightarrow \frac{\sin \hat{B}}{1\omega\sqrt{r}} = \frac{1}{r}$$

$$\hat{B} = 80^\circ \Rightarrow \frac{80^\circ}{180^\circ} = \frac{R}{\pi} \Rightarrow \boxed{R = \frac{\pi}{2}}$$

$$\hat{C} = 100^\circ \Rightarrow \frac{100^\circ}{180^\circ} = \frac{R}{\pi} \Rightarrow R = \frac{100^\circ \pi}{180^\circ} = \frac{5\pi}{9}$$

$$\Rightarrow \hat{B} = 80^\circ$$

$$\sin \hat{B} = \frac{1\omega\sqrt{r}}{r} = \frac{\sqrt{r}}{r}$$

$$\Rightarrow \hat{B} = 80^\circ \Rightarrow \hat{C} = 100^\circ$$

γ)

$$\frac{\tan(\pi - \alpha) + r \tan(\pi + \alpha)}{\tan(\pi - \alpha) - \tan(\pi + \alpha)} = \frac{-\tan \alpha + r + \tan \alpha}{-\tan \alpha - \tan \alpha}$$

$$\tan(\pi - \alpha) = \tan(\pi + \alpha)$$

$$= \frac{r \tan \alpha}{-r \tan \alpha} = -1$$

✓)

$$\tan \frac{\pi}{12} = \alpha$$

$$1.4 \frac{\pi}{12} = \frac{1}{\alpha}$$

$$\frac{\pi}{12}$$

$$\frac{\pi}{12} = \frac{D}{180^\circ} \Rightarrow D = 10^\circ$$

$$\Rightarrow \frac{r \tan 10^\circ + \tan 10^\circ}{r \tan 10^\circ - \tan 10^\circ}$$

$$= \frac{r \tan(\frac{\pi}{12} - 10^\circ) + \tan(\frac{\pi}{12} + 10^\circ)}{r \tan(\pi - 10^\circ) - \tan(\frac{\pi}{12} - 10^\circ)}$$

$$r \tan(\pi - 10^\circ) = \tan(\frac{\pi}{12} - 10^\circ)$$

$$= \frac{r(1 + 10^\circ) + (-1 + 10^\circ)}{r \tan 10^\circ - (-1 + 10^\circ)} = \frac{\frac{r}{\alpha} - \frac{1}{\alpha}}{-r\alpha - \frac{1}{\alpha}} = \frac{\frac{1}{\alpha}}{-\frac{r\alpha^2 + 1}{\alpha}} = \frac{1}{-r\alpha^2 - 1}$$

$$1) = \frac{\sin x + \cos x}{\sin x - \cos x} + \frac{\sin x - \cos x}{\sin x + \cos x} = r$$

حرف مشترک
 $\Rightarrow$

$$\frac{(\sin x + \cos x)^2 + (\sin x - \cos x)^2}{(\sin x - \cos x)(\sin x + \cos x)} = \frac{\sin^2 x + \cos^2 x + \sin^2 x + \cos^2 x}{\sin^2 x - \cos^2 x}$$

$$\frac{2\sin^2 x + 2\cos^2 x}{\sin^2 x - \cos^2 x} = \frac{2}{\sin^2 x - \cos^2 x} = r$$

$$\Rightarrow 2\sin^2 x - 2\cos^2 x = r$$

$$\sin^2 x - \cos^2 x = \frac{r}{2}$$

$$\sin^2 x = \cos^2 x + \frac{r}{2}$$

$$\cancel{\sin^2 x} - \cancel{\cos^2 x} = \frac{r}{2}$$

$$\Rightarrow \sin^2 x + \cos^2 x = 1$$

$$\cos^2 x + \cos^2 x + \frac{r}{2} = 1$$

$$\Rightarrow 2\cos^2 x = \frac{1}{2}$$

$$\cos^2 x = \frac{1}{4} \Rightarrow \sin^2 x = \frac{3}{4}$$

$$\Rightarrow \tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{\frac{3}{4}}{\frac{1}{4}} = 3$$

$$= 3$$

$$9) \frac{\sin^2 x - 2\cos^2 x + 1}{\sin^2 x + 2\cos^2 x - 1} = 2$$

$$1 = \sin^2 x + \cos^2 x \Rightarrow \frac{\sin^2 x - 2\cos^2 x + \sin^2 x + \cos^2 x}{\sin^2 x + 2\cos^2 x - \sin^2 x - \cos^2 x} = 2$$

$$\sin^2 x + \cos^2 x = 1$$

$$2(1 - \cos^2 x) = 1$$

$$\cancel{2\sin^2 x} = 1$$

$$\sin^2 x = \frac{1}{2} \Rightarrow \tan^2 x = \frac{\frac{1}{2}}{\frac{1}{2}} = 1$$

$$\frac{2\sin^2 x - 2\cos^2 x}{\cos^2 x} = 2 \Rightarrow 2\cos^2 x \sin^2 x - 2\cos^4 x = 2\cos^2 x$$

$$\Rightarrow 2\cos^2 x = 2\sin^2 x$$

$$\Rightarrow \sin^2 x = \cos^2 x$$

۱۵۱

$$\cos(2\alpha) = \frac{\sqrt{2+\sqrt{2}}}{2}$$

$$\begin{aligned} \cos^2 \alpha &= \frac{1 + \cos(2\alpha)}{2} \xrightarrow{\text{برای ساده کردن}} \frac{1 + \frac{\sqrt{2}}{2}}{2} = \frac{2 + \sqrt{2}}{4} = \frac{2 + \sqrt{2}}{4} \cdot \frac{2 + \sqrt{2}}{2 + \sqrt{2}} \\ &= \cos^2(45^\circ) = \cos^2(45^\circ) = \frac{\sqrt{2+\sqrt{2}}}{2} \end{aligned}$$

$$\sin(45^\circ) = \frac{\sqrt{2+\sqrt{2}}}{2}$$

$$\sin^2 \alpha = \frac{1 - \cos(2\alpha)}{2} \xrightarrow{\text{برای ساده کردن}} \frac{1 - \frac{\sqrt{2}}{2}}{2} = \frac{2 - \sqrt{2}}{4}$$

$$= \sin^2(45^\circ) =$$

$$\sin(45^\circ) = \frac{\sqrt{2+\sqrt{2}}}{2}$$

$$\text{راه دوم: } \underbrace{\sin(45^\circ)}_{\text{مستقیم میگیریم}} = \cos(45^\circ) = \frac{\sqrt{2+\sqrt{2}}}{2}$$