۲۰ عالی نوشتی!

نام و نام خانوادگی ماهان بهشتی اللهی پاسخنامه تشریحی تکلیف شماره ۱۸ کلاس تجربی A شنبه‌ها

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الف) $\tan\frac{11\pi}{4} + \sin\frac{15\pi}{4} \cdot \cos\frac{13\pi}{4}$

$\frac{11\pi}{4} = 2\pi + \frac{3\pi}{4} \Rightarrow \tan\left(\frac{11\pi}{4}\right) \Rightarrow \tan\left(\frac{3\pi}{4}\right) = -1 \qquad -\frac{\sqrt{2}}{2} \times -\frac{\sqrt{2}}{2} = \frac{1}{2} + (-1) = -\frac{1}{2}$ (۲)

$\sin\left(\frac{15\pi}{4}\right) \Rightarrow \sin\left(\frac{7\pi}{4}\right) = -\frac{\sqrt{2}}{2}, \quad \cos\left(\frac{13\pi}{4}\right) \Rightarrow \cos\left(\frac{5\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

ب) $\tan\frac{17\pi}{4} \times \sin\frac{11\pi}{3} + \cos\frac{10\pi}{3}$

$\tan\frac{17\pi}{4} \Rightarrow \tan\frac{5\pi}{6} = -\frac{\sqrt{3}}{3}, \quad \sin\frac{11\pi}{3} \Rightarrow \sin\frac{5\pi}{3} = -\frac{\sqrt{3}}{2}$

$\cos\frac{10\pi}{3} \Rightarrow \cos\frac{4\pi}{3} = -\frac{1}{2}$

$\Big\} \; -\frac{\sqrt{3}}{3} \times -\frac{\sqrt{3}}{2} = \frac{1}{2} + \left(-\frac{1}{2}\right) = 0$

۲

$\cot x = 4 \qquad \frac{\cos x}{\sin x} = 4 \rightsquigarrow \cos x = 4K, \; \sin x = K$ (۲)

$\frac{3\cos x - \sin x}{\cos x + \sin x} = \frac{3(4K) - K}{4K + K} = \frac{11K}{5K} = \frac{11}{5}$

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$\sin\alpha = 2\cos\alpha \Rightarrow \sin\alpha = 2c, \quad \cos\alpha = c$

$\sin^2\alpha + \cos^2\alpha = 1 \Rightarrow (2c)^2 + c^2 = 1 \Rightarrow 4c^2 + c^2 = 1 \Rightarrow 5c^2 = 1 \Rightarrow c^2 = \frac{1}{5}$ (۲)

$\Rightarrow c = \pm\frac{1}{\sqrt{5}}, \quad \cos\alpha < 0 \Longrightarrow \cos\alpha = -\frac{1}{\sqrt{5}}$

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الف)

$\cos\left(\frac{11\pi}{4} + \alpha\right) = \cos\left(\frac{3\pi}{4} + \alpha\right) = \cos\frac{3\pi}{4}\cos\alpha - \sin\frac{3\pi}{4}\sin\alpha$

$\cos\alpha = -\sqrt{1 - \sin^2\alpha} = -\sqrt{1 - \frac{2}{100}} = -\frac{7\sqrt{2}}{10}$ (۲)

$\cos\left(\frac{11\pi}{4} + \alpha\right) = -\frac{\sqrt{2}}{2}\cos\alpha - \frac{\sqrt{2}}{2}\sin\alpha = -\frac{\sqrt{2}}{2}(\cos\alpha + \sin\alpha)$.

$\cos\alpha + \sin\alpha = -\frac{7\sqrt{2}}{10} + \frac{\sqrt{2}}{10} = -\frac{6\sqrt{2}}{10} = -\frac{3\sqrt{2}}{5} \Rightarrow \cos\left(\frac{11\pi}{4} + \alpha\right) = -\frac{\sqrt{2}}{2}\left(-\frac{3\sqrt{2}}{5}\right) = \frac{3}{5}$

ب)

$\sin\left(\frac{13\pi}{4} + \alpha\right) \Rightarrow \sin\left(3\pi + \frac{\pi}{4} + \alpha\right) \Rightarrow -\sin\left(\alpha + \frac{\pi}{4}\right)$

$\sin\alpha = \frac{1}{\sqrt{1+7^2}} = \frac{1}{\sqrt{50}} = \frac{1}{5\sqrt{2}}, \quad \cos\alpha = \frac{7}{5\sqrt{2}} \Rightarrow \sin\left(\alpha + \frac{\pi}{4}\right) = \sin\alpha\cos\frac{\pi}{4} + \cos\alpha\sin\frac{\pi}{4}$

$\sin\frac{\pi}{4} = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2} \Rightarrow \sin\left(\alpha + \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}(\sin\alpha + \cos\alpha) = \sin\left(\alpha + \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} \times \frac{8}{5\sqrt{2}} = \frac{8}{10} = \frac{4}{5} \Rightarrow -\frac{4}{5}$

جواب نهایی

~~$\sin\left(\frac{13\pi}{4} + \alpha\right) = -\frac{4}{5}$~~ غلط جواب سمت ب

۵

$2\sin^2 x + \cos^2 x = \frac{4}{3} \Rightarrow \cos^2 x = 1 - \sin^2 x \Rightarrow 2\sin^2 x + (1 - \sin^2 x) = \frac{4}{3}$ (۲)

$\sin^2 x + 1 = \frac{4}{3} \Rightarrow \sin^2 x = \frac{4}{3} - 1 = \frac{1}{3} \Rightarrow \sin^2 x = \frac{1}{3} \Rightarrow \cos^2 x = 1 - \sin^2 x = 1 - \frac{1}{3} = \frac{2}{3}$

$\tan^2 x = \frac{\sin^2 x}{\cos^2 x} \Rightarrow \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$

۶

$a=\sqrt{3}\ ,\ b=6\ ,\ S=4.5$

$S=\frac{1}{2}ab\sin\alpha \Rightarrow 4.5=\frac{1}{2}(\sqrt{3}\times 6)\sin\alpha = 3\sqrt{3}\sin\alpha = 4.5$

$\sin\alpha = \frac{4.5}{3\sqrt{3}} = \frac{\sqrt{3}}{2} \Rightarrow \sin\alpha = \frac{\sqrt{3}}{2} \Rightarrow \alpha = \frac{\pi}{3}$ یا $\alpha = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$\alpha_{min} = \frac{\pi}{3}\ ,\ \alpha_{max} = \frac{2\pi}{3} \Longrightarrow \frac{\alpha_{max}}{\alpha_{min}} = \frac{\frac{2\pi}{3}}{\frac{\pi}{3}} = 2$

(۲)

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$\left.\begin{array}{l} S=54 \\ \frac{a}{b}=\frac{2}{3} \\ \alpha=150^\circ \end{array}\right\}\ \begin{array}{l} a=2x \\ b=3x \end{array} \Rightarrow S=ab\sin\alpha \Rightarrow 54=(2x)(3x)\times\frac{1}{2} \Rightarrow x^2=18 \Rightarrow x=\sqrt{18} \Rightarrow x=3\sqrt{2}$

$a=2x=6\sqrt{2}\ ,\ b=3x=9\sqrt{2}$

$P=2(a+b)=2(6\sqrt{2}+9\sqrt{2})=2(15\sqrt{2})=\boxed{30\sqrt{2}}$

(۲)

۸

$\left.\begin{array}{l} S_{A\hat{B}C} = \frac{1}{2}\times 5\times 7\times\sin\hat{A} = 17.5\sin\hat{A} \\ S_{A\hat{D}E} = \frac{1}{2}\times 7\times 4\times\sin\hat{A} = 14\sin\hat{A} \end{array}\right\}\ 3.5\sin\hat{A}=1.75$

$\Downarrow$

$\sin\hat{A} = \frac{1.75}{3.5} = 0.5$

$\Downarrow$

$\hat{A} = 30^\circ$

$\tan 30^\circ = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{\cancel{2}}{\cancel{2}\sqrt{3}} = \frac{1}{\sqrt{3}}$

(۲)

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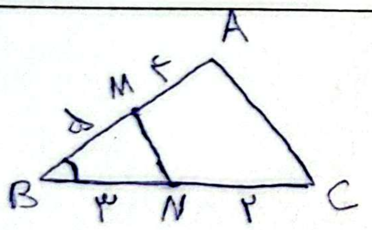

$2\overline{BN} = 3\overline{NC} \Longrightarrow \frac{\overline{BN}}{\overline{NC}} = \frac{3}{2} \Rightarrow \overline{BN}=3,\ \overline{NC}=2$

$S_{A\hat{B}C} = 3\,S_{B\hat{M}N}$

$S_{\text{مثلث}} = \frac{1}{2}ab\times bc\times\sin\hat{B}$ فرمول مساحت مثلث

$S_{A\hat{B}C} = \frac{1}{2}\times 5\times AB \Rightarrow 2.5AB \to \boxed{9=AB}$ $\quad AB=AM+BM=4K+5K=9K$

$S_{B\hat{M}N} = \frac{1}{2}\times 3\times BM \Rightarrow 1.5BM \to \boxed{5=BM} \Longrightarrow \frac{BM}{AB} = \frac{5}{9}$

(۲)

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$\frac{1}{|\cos\alpha|} - \tan\alpha = \frac{1+\sin\alpha}{|\cos\alpha|} \Rightarrow -\tan\alpha = \frac{\sin\alpha}{|\cos\alpha|}$

$\begin{cases} \cos\alpha>0 \Rightarrow -\tan\alpha = \frac{\sin\alpha}{\cos\alpha} \Rightarrow -\sin\alpha/\cos\alpha = \sin\alpha/\cos\alpha \Rightarrow \sin\alpha=0 \\ \cos\alpha<0 \Rightarrow -\tan\alpha = \frac{\sin\alpha}{-\cos\alpha} \Rightarrow -\tan\alpha = -\tan\alpha \Rightarrow \end{cases}$ همیشه درست است

$\cos\alpha>0 \Rightarrow \sin\alpha=0 \Rightarrow \alpha=0^\circ, 360^\circ$

$\cos\alpha<0 \Rightarrow 90^\circ<\alpha<270^\circ$

$\frac{|\sin\alpha|}{\cos\alpha} = -\frac{1}{\cot\alpha} \Longrightarrow \cot\alpha = \frac{\cos\alpha}{\sin\alpha} \Longrightarrow \frac{1}{\cot\alpha} = \frac{\sin\alpha}{\cos\alpha} \Rightarrow \frac{|\sin\alpha|}{\cos\alpha} = \frac{-\sin\alpha}{\cos\alpha}$

$|\sin\alpha| = -\sin\alpha \Rightarrow \sin\alpha\leq 0$ باتوجه به حل دو معادله انتهای کمان α در ناحیه‌ی چهارم قرار

می‌گیرد بجز نقطه‌ی مرزی

(۲)