نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ..... کلاس .....A.....

۱

الف) $\tan\frac{3\pi}{4}+\sin\frac{7\pi}{4}\cdot\cos\frac{8\pi}{4}=\tan 135^\circ+\sin 315^\circ\cdot\cos 360^\circ=-1+\left(-\frac{\sqrt{2}}{2}\right)\times\left(-\frac{\sqrt{2}}{2}\right)=-1+\frac{1}{2}=-\frac{1}{2}$

ب) $\tan\frac{8\pi}{6}\cdot\sin\frac{9\pi}{3}+\cos\frac{4\pi}{3}=\tan 150^\circ\cdot\sin 300^\circ+\cos 240^\circ=-\frac{\sqrt{3}}{3}\times\left(-\frac{\sqrt{3}}{2}\right)-\frac{1}{2}=\frac{1}{2}-\frac{1}{2}=0$

۲

$\cot^2x+1=\frac{1}{\sin^2x}\Rightarrow 4^2+1=\frac{1}{\sin^2x}\Rightarrow\sin^2x=\frac{1}{17}\Rightarrow\sin x=\pm\frac{\sqrt{17}}{17}$

$\sin^2x+\cos^2x=1\Rightarrow\cos^2x=1-\sin^2x=1-\frac{1}{17}=\frac{16}{17}\Rightarrow\cos x=\pm\frac{4\sqrt{17}}{17}$

$\cot x>0$ و $\cos x,\sin x<0$ یا $\cos x,\sin x>0\Longleftarrow\cos x=4\sin x$

$\frac{3\cos x-\sin x}{\cos x+\sin x}=\frac{12\sin x-\sin x}{4\sin x+\sin x}=\frac{11\sin x}{5\sin x}=\frac{11}{5}$

۳

$\sin^2\alpha+\cos^2\alpha=1\Rightarrow\underbrace{4\cos^2\alpha+\cos^2\alpha}_{5\cos^2\alpha}=1\Rightarrow\cos^2\alpha=\frac{1}{5}\xrightarrow{\cos\alpha<0}\cos\alpha=-\frac{\sqrt{5}}{5}$

۴

الف) $\cos^2\alpha=1-\sin^2\alpha=1-\frac{2}{100}=\frac{98}{100}\Rightarrow\cos\alpha\xrightarrow[\cos\alpha<0]{\text{ربع سوم}}-\frac{7\sqrt{2}}{10}$

$\cos 135^\circ\cdot\cos\alpha-\sin 135^\circ\cdot\sin\alpha=-\frac{\sqrt{2}}{2}\times-\frac{7\sqrt{2}}{10}-\frac{\sqrt{2}}{2}\times\frac{\sqrt{2}}{10}=\frac{14}{20}-\frac{2}{20}=\frac{12}{20}=\frac{3}{5}$

ب) $\tan^2\alpha+1=\frac{1}{\cos^2\alpha}\Rightarrow\frac{50}{49}=\frac{1}{\cos^2\alpha}\Rightarrow\cos\alpha\xrightarrow{\text{ربع سوم}}-\frac{7\sqrt{2}}{10}$

$\tan\alpha=\frac{\sin\alpha}{\cos\alpha}=\frac{1}{7}=\frac{\sin\alpha}{-\frac{7\sqrt{2}}{10}}\Rightarrow\sin\alpha=-\frac{\sqrt{2}}{10}$

$\sin 225^\circ\cdot\cos\alpha+\sin\alpha\cdot\cos 225^\circ=-\frac{\sqrt{2}}{2}\times-\frac{7\sqrt{2}}{10}+\frac{\sqrt{2}}{10}\times\left(-\frac{\sqrt{2}}{2}\right)=\frac{14}{20}-\frac{2}{20}=\frac{12}{20}=\frac{3}{5}$

۵

$\sin^2x+\underbrace{\sin^2x+\cos^2x}_{1}=\frac{4}{3}\Rightarrow\sin^2x=\frac{1}{3}$

$\cos^2x=1-\sin^2x=1-\frac{1}{3}=\frac{2}{3}$ $\qquad$ $\tan^2x+1=\frac{1}{\cos^2x}\Rightarrow\tan^2x=\frac{1}{\frac{2}{3}}-1\Rightarrow$

$\tan^2x=\frac{3}{2}-1=\frac{1}{2}$

۶

$$S_{ABC}^{\triangle} = \frac{1}{2} ab \cdot \sin\alpha \Rightarrow 6\sqrt{3} = \frac{1}{2} \sqrt{3} \times 4\sqrt{3} \times \sin\alpha \Rightarrow \sin\alpha = \frac{6\sqrt{3}}{3\sqrt{3}} = \frac{\sqrt{3}}{2}$$

$$\sin\alpha = \frac{\frac{\sqrt{3}}{2}}{\sqrt{3}} = \frac{\frac{\sqrt{3}}{2}}{2\sqrt{3}} = \frac{\sqrt{3}}{2} \begin{cases} \alpha = 60^\circ \rightarrow \min\alpha \\ \alpha = 120^\circ \rightarrow \max\alpha \end{cases}$$

$$\frac{\max\alpha}{\min\alpha} = \frac{120}{60} = 2$$

۷

$$S_{\text{متوازی‌الاضلاع}} = ab \cdot \sin\alpha \rightarrow 36 = 2x \cdot 3x \cdot \sin 30^\circ = 6x^2 \cdot \frac{1}{2} \Rightarrow$$

($2x$, $3x$, 30°)

$$3x^2 = 36 \Rightarrow x^2 = 18 \xrightarrow{x>0} x = \sqrt{18} = 3\sqrt{2}$$

$$a = 2x = 6\sqrt{2}$$

$$b = 3x = 9\sqrt{2}$$

$$P_{\text{متوازی‌الاضلاع}} = 2(a+b) = 2(\underbrace{6\sqrt{2} + 9\sqrt{2}}_{15\sqrt{2}}) = 30\sqrt{2}$$

۸

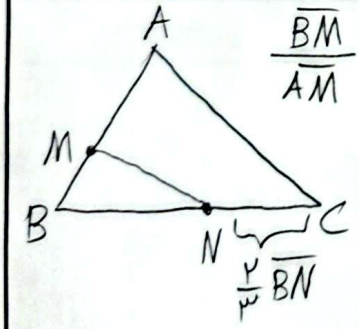
$$11\sqrt{3} = S_{ABC}^{\triangle} - S_{ADE}^{\triangle} = \frac{1}{2} AB \cdot AC \cdot \sin\hat{A} - \frac{1}{2} AE \cdot AD \cdot \sin\hat{A} \Rightarrow$$

$$11\sqrt{3} = \underbrace{\frac{1}{2} \times 10 \times 7 \times \sin\hat{A}}_{17.5\sin\hat{A}} - \underbrace{\frac{1}{2} \times 4 \times 7 \times \sin\hat{A}}_{14\sin\hat{A}} \Rightarrow 11\sqrt{3} = 3.5\sin\hat{A} \Rightarrow \sin\hat{A} = \frac{1}{2}$$

$$\hat{A} = 60^\circ \quad (\hat{A} < 90^\circ)$$

$$\tan\hat{A} = \tan 60^\circ = \sqrt{3}$$

۹



$$\frac{BM}{AM} = ? \qquad 2\overline{BN} = 3\overline{NC} \Rightarrow \overline{NC} = \frac{2}{3}\overline{BN}$$

$$3 = \frac{S_{ABC}^{\triangle}}{S_{BMN}^{\triangle}} = \frac{\frac{1}{2} AB \cdot BC \cdot \sin\hat{B}}{\frac{1}{2} \cdot BM \cdot BN \cdot \sin\hat{B}} \Rightarrow 3 = \frac{(AM+BM)\frac{5}{3}BN}{BM \cdot BN} \Rightarrow$$

$$3BM = \frac{5}{3} AM + \frac{5}{3} BM \Rightarrow \frac{4}{3} BM = \frac{5}{3} AM \Rightarrow \frac{BM}{AM} = \frac{5}{4}$$

۱۰

$$\frac{1}{\sqrt{\cos^2\alpha}} - \tan\alpha = \frac{1+\sin\alpha}{|\cos\alpha|} \Rightarrow \frac{1}{|\cos\alpha|} - \tan\alpha = \frac{1}{|\cos\alpha|} + \frac{\sin\alpha}{|\cos\alpha|} \Rightarrow -\tan\alpha = \frac{\sin\alpha}{|\cos\alpha|} \quad (1)$$

$$\frac{|\sin\alpha|}{\cos\alpha} = -\frac{1}{\cot\alpha} = -\tan\alpha \quad (2)$$

$$(1) \wedge (2) \Rightarrow -\tan\alpha = \frac{|\sin\alpha|}{\cos\alpha} = \frac{\sin\alpha}{|\cos\alpha|}$$

$$\Leftarrow \begin{cases} -\frac{\sin\alpha}{\cos\alpha} = \frac{|\sin\alpha|}{\cos\alpha} \Rightarrow \sin\alpha < 0 \\ -\frac{\sin\alpha}{\cos\alpha} = \frac{\sin\alpha}{|\cos\alpha|} \Rightarrow \cos\alpha < 0 \end{cases}$$

انتهای کمان α در ناحیه سوم است