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$$\tan \frac{11\pi}{4} + \sin \frac{10\pi}{4} \cos \frac{13\pi}{4} =$$

$$\frac{11\pi}{4} = 2\pi + \frac{3\pi}{4} \Rightarrow \tan \frac{11\pi}{4} = \tan \frac{3\pi}{4}$$

$$\frac{3\pi}{4} = \pi - \frac{\pi}{4} \Rightarrow \tan \frac{3\pi}{4} = -\tan \frac{\pi}{4} = -1$$

$$\frac{10\pi}{4} = 2\pi - \frac{\pi}{2} \quad \sin \frac{10\pi}{4} = \sin (2(2\pi) - \frac{\pi}{2}) = -\sin \frac{\pi}{2} = -\frac{\sqrt{2}}{2}$$

$$\cos \frac{13\pi}{4} = \cos \frac{5\pi}{4} \Rightarrow \cos (\pi + \frac{\pi}{4}) = -\cos \frac{\pi}{4} = -\frac{\sqrt{2}}{2}$$

$$-1 + (-\frac{\sqrt{2}}{2}) (-\frac{\sqrt{2}}{2}) = -1 + \frac{2}{4} = -1 + \frac{1}{2} = -\frac{1}{2}$$

$$\tan \frac{13\pi}{4} \sin \frac{12\pi}{4} + \cos \frac{10\pi}{4} = \tan \frac{5\pi}{4} = -\frac{\sqrt{2}}{2}$$

$$\cos \frac{10\pi}{4} = -\frac{1}{2} \quad \sin \frac{12\pi}{4} = \sin (3\pi - \frac{\pi}{2}) = -\sin \frac{\pi}{2} = -\frac{\sqrt{2}}{2}$$

$$(-\frac{\sqrt{2}}{2})(-\frac{\sqrt{2}}{2}) + (-\frac{1}{2}) = \frac{2}{4} + (-\frac{1}{2}) = 0$$

$$\frac{2\cos x - \sin x}{\cos x + \sin x} = \cot x = \frac{\cos x}{\sin x} \Rightarrow \frac{2\cos x - \sin x}{\frac{\cos x}{\sin x} + \frac{\sin x}{\sin x}} = \frac{2\cos x - \sin x}{\frac{\cos x + \sin x}{\sin x}} = \frac{(2\cos x - \sin x)\sin x}{\cos x + \sin x} = 2\cos x - 1$$

$$= \frac{2\cos x - 1}{\cos x + 1} = \frac{2 \times \frac{1}{2} - 1}{\frac{1}{2} + 1} = \frac{1 - 1}{\frac{3}{2}} = 0$$

$$\frac{\sin \alpha}{\cos \alpha} = 2 \quad \tan \alpha = 2 \quad \frac{\sin^2 \alpha}{\cos^2 \alpha} + \frac{\cos^2 \alpha}{\cos^2 \alpha} = 1$$

$$\tan^2 \alpha + 1 = \frac{1}{\cos^2 \alpha} \xrightarrow{\tan \alpha = 2} 2^2 + 1 = \frac{1}{\cos^2 \alpha} \Rightarrow \cos^2 \alpha = \frac{1}{5}$$

$$\cos \alpha = \pm \frac{\sqrt{5}}{5} \xrightarrow{\text{چون در ربع سوم است}} -\frac{\sqrt{5}}{5}$$

$$\cos (\frac{11\pi}{4} + \alpha) = -\frac{\sqrt{2}}{2} (\cos \alpha + \sin \alpha) \Rightarrow \cos^2 \alpha = 1 - \frac{2}{100} = \frac{98}{100}$$

$$1 - \frac{1}{50} = \frac{49}{50} \quad \cos \alpha = -\sqrt{\frac{49}{50}} = -\frac{7}{\sqrt{50}}$$

$$\cos \alpha + \sin \alpha = (-\frac{7\sqrt{2}}{10}) + (\frac{\sqrt{2}}{10}) = -\frac{6\sqrt{2}}{10} \Rightarrow -\frac{\sqrt{2}}{2} (-\frac{3\sqrt{2}}{5})$$

$$= \frac{2 \times (\sqrt{2} \times \sqrt{2})}{10} = \frac{2 \times 2}{10} = \frac{4}{10} = \frac{2}{5}$$

$$\frac{13\pi}{4} = 2\pi + \frac{5\pi}{4} \Rightarrow \sin (\frac{13\pi}{4} + \alpha) = -\sin (\frac{5\pi}{4} + \alpha) =$$

$$1 + \tan^2 \alpha = \frac{1}{\cos^2 \alpha} \Rightarrow 1 + (\frac{1}{\sqrt{2}})^2 = \frac{1}{\cos^2 \alpha} \Rightarrow \frac{3}{2} = \frac{1}{\cos^2 \alpha} \Rightarrow \cos^2 \alpha = \frac{2}{3}$$

$$\cos \alpha = \sqrt{\frac{2}{3}} = \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{2}\sqrt{3}}{10} \quad \sin \alpha = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{10}$$

$$\sin\left(\frac{\pi}{4} + \alpha\right) = \sin\left(\frac{\pi}{4}\right) \cos \alpha + \cos\left(\frac{\pi}{4}\right) \sin \alpha \Rightarrow \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2} \Rightarrow \sin\left(\frac{\pi}{4} + \alpha\right) = \frac{\sqrt{2}}{2} \left(\frac{\sqrt{2}}{2}\right) = \frac{2 \times (\sqrt{2} \times \sqrt{2})}{10} = \frac{4}{10} = \frac{2}{5}$$

$$\sin^2 x + (\sin^2 x + \cos^2 x) = \frac{5}{3} \quad \sin^2 x + \cos^2 x = 1 \Rightarrow \sin^2 x + 1 = \frac{5}{3} \Rightarrow \sin^2 x = \frac{2}{3}$$

$$\sin^2 x = \frac{2}{3} \Rightarrow 1 - \cos^2 x = \frac{2}{3} \Rightarrow \cos^2 x = 1 - \frac{2}{3} = \frac{1}{3} \Rightarrow \cos x = \pm \frac{1}{\sqrt{3}}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{\frac{\sqrt{2}}{\sqrt{3}}}{\frac{1}{\sqrt{3}}} = \sqrt{2}$$

$$S = \frac{1}{2} a b \sin \alpha \Rightarrow \frac{5}{3} = \frac{1}{2} \times 9 \times \sqrt{2} \times \sin \alpha$$

$$\frac{5}{3} = \frac{9\sqrt{2}}{2} \sin \alpha \Rightarrow \sin \alpha = \frac{5}{3} \times \frac{2}{9\sqrt{2}} = \frac{\sqrt{2}}{3}$$

$$\alpha_{\min} = 40^\circ \quad 140 - 40 = 100 \quad \frac{140^\circ}{50^\circ} = 2.8$$

$$\frac{a}{b} = \frac{x}{y} \quad a = 2x \quad b = 3x \quad 100 - 100 = 0$$

$$\sin 100 = \sin(100 - 0) = \sin 0 = \frac{1}{y}$$

$$\omega f = (2\pi) \times (3x) \times \sin(100) \Rightarrow \omega f = 6\pi x \times \frac{1}{y} \Rightarrow \omega f = \frac{6\pi x}{y}$$

$$x^2 = \frac{\omega f}{2\pi} = 1 \quad x = \sqrt{2} \quad a = 2 \times \sqrt{2} = 2\sqrt{2} \quad b = 3 \times \sqrt{2} = 3\sqrt{2}$$

$$P = 2(a+b) \Rightarrow 2(2\sqrt{2} + 3\sqrt{2}) = 10\sqrt{2}$$

$$S_{ABC} = \frac{1}{2} \times AB \times AC \times \sin(C) \quad S_{ADE} = \frac{1}{2} \times AD \times AE \times \sin(C)$$

$$|S_{ABC} - S_{ADE}| = \frac{1}{2} \times 10 \times \sin(C) - \frac{1}{2} \times 1 \times \sin(C) = \frac{9}{2} \sin(C) = \frac{1}{2} \Rightarrow \sin(C) = \frac{1}{9} \Rightarrow C = \arcsin\left(\frac{1}{9}\right)$$

$$BN = NC \Rightarrow BN = \frac{1}{2} BC \quad BC = BN + NC = \frac{1}{2} BC + \frac{1}{2} BC = BC$$

$$\frac{BN}{BC} = \frac{1}{2} \quad \frac{S_{BMN}}{S_{ABC}} = \frac{BM \times BN}{AB \times BC} = \frac{1}{4} \quad \frac{S_{BMN}}{S_{ABC}} = \frac{1}{4}$$

$$\frac{BM}{AB} = \frac{1}{4} \quad \frac{1}{4} \times \frac{1}{2} = \frac{1}{8} \quad \frac{BM}{AB} = \frac{1}{8} \quad AM = AB - BM = AB \left(1 - \frac{1}{8}\right) = \frac{7}{8} AB$$

$$\frac{1}{\cos \alpha} - \tan \alpha = \frac{1 + \sin \alpha}{1 \cos \alpha} \Rightarrow -\tan \alpha = \frac{1 + \sin \alpha - 1}{1 \cos \alpha} \Rightarrow \tan \alpha = \frac{\sin \alpha}{\cos \alpha}$$

$$-\frac{\sin \alpha}{\cos \alpha} = \frac{\sin \alpha}{\cos \alpha} \quad \sin \neq 0 \Rightarrow \frac{1}{\cos \alpha} = \frac{1}{\cos \alpha} \quad \frac{1}{\cos \alpha} + \frac{1}{\cos \alpha} = 0$$