نام و نام خانوادگی: علی پورعابدین   کلاس: دهم سری A شماره ۱۸   ۲۰ آفرین!

۱- الف)

$\tan\frac{11\pi}{4}+\sin\frac{15\pi}{4}\cos\frac{13\pi}{4}=$

$\frac{11\pi}{4}=2\pi+\frac{3\pi}{4} \Rightarrow \tan\frac{11\pi}{4}=\tan\frac{3\pi}{4}$ (۲)

$\frac{3\pi}{4}=\pi-\frac{\pi}{4} \Rightarrow \tan\frac{3\pi}{4}=-\tan\frac{\pi}{4}=-1$

$\frac{15\pi}{4}=4\pi-\frac{\pi}{4}$ $\sin\frac{15\pi}{4}=\sin(2(2\pi)-\frac{\pi}{4})=-\sin\frac{\pi}{4}=-\frac{\sqrt{2}}{2}$

$\cos\frac{13\pi}{4}=\cos\frac{5\pi}{4} \Rightarrow \cos(\pi+\frac{\pi}{4})=-\cos\frac{\pi}{4}=-\frac{\sqrt{2}}{2}$

$-1+(-\frac{\sqrt{2}}{2})(-\frac{\sqrt{2}}{2})=-1+\frac{2}{4}=-1+\frac{1}{2}=-\frac{1}{2}$ ✓

ب)

$\tan\frac{17\pi}{6}\sin\frac{11\pi}{3}+\cos\frac{10\pi}{3}=$ $\tan\frac{17\pi}{6}=-\frac{\sqrt{3}}{3}$

$\cos\frac{10\pi}{3}=-\frac{1}{2}$ $\sin\frac{11\pi}{3}=\sin(4\pi-\frac{\pi}{3})=-\sin\frac{\pi}{3}=-\frac{\sqrt{3}}{2}$

$(-\frac{\sqrt{3}}{3})(-\frac{\sqrt{3}}{2})+(-\frac{1}{2})=\frac{1}{2}+(-\frac{1}{2})=0$ ✓

۲-

$\frac{3\cos x-\sin x}{\cos x+\sin x}=\cot x=\frac{4\cos x}{\sin x} \Rightarrow \frac{\frac{3\cos x}{\sin x}-\frac{\sin x}{\sin x}}{\frac{\cos x}{\sin x}+\frac{\sin x}{\sin x}}=\frac{3(\frac{\cos x}{\sin x})-1}{(\frac{\cos x}{\sin x})+1}$ (۲)

$=\frac{3\cot x-1}{\cot x+1}=\frac{3\times 4-1}{4+1}=\frac{11}{5}$ ✓

۳-

$\frac{\sin\alpha}{\cos\alpha}=2$ $\tan=2$ $\frac{\sin^2\alpha}{\cos^2\alpha}+\frac{\cos^2\alpha}{\cos^2\alpha}=\frac{1}{\cos^2\alpha}$ (۲)

$\tan^2\alpha+1=\frac{1}{\cos^2\alpha} \xrightarrow{\tan=2} 2^2+1=\frac{1}{\cos^2\alpha} \Rightarrow \cos^2\alpha=\frac{1}{5}$

$\cos\alpha=\pm\frac{\sqrt{5}}{5}$ $\xrightarrow{\text{چون در ربع سوم است}}$ $-\frac{\sqrt{5}}{5}$ ✓

۴- الف)

$\cos(\frac{11\pi}{4}+\alpha)=-\frac{\sqrt{2}}{2}(\cos\alpha+\sin\alpha) \Rightarrow \cos^2\alpha=1-\frac{2}{100}=$ (۲)

$1-\frac{1}{50}=\frac{49}{50}$ چون α در ربع ۲ است $\Rightarrow \cos\alpha=-\sqrt{\frac{49}{50}}=-\frac{7}{\sqrt{50}}$

$\cos\alpha+\sin\alpha=(-\frac{7\sqrt{2}}{10})+(\frac{\sqrt{2}}{10})=-\frac{3\sqrt{2}}{5} \Rightarrow -\frac{\sqrt{2}}{2}(-\frac{3\sqrt{2}}{5})$

$=\frac{3\times(\sqrt{2}\times\sqrt{2})}{10}=\frac{3\times 2}{10}=\frac{6}{10}=\frac{3}{5}$ ✓

ب)

$\frac{13\pi}{4}=3\pi+\frac{\pi}{4} \Rightarrow \sin(\frac{13\pi}{4}+\alpha)=-\sin(\frac{\pi}{4}+\alpha) \Rightarrow$

$1+\tan^2\alpha=\frac{1}{\cos^2\alpha} \Rightarrow 1+(\frac{1}{7})^2=\frac{1}{\cos^2\alpha} \Rightarrow \frac{49}{50} \Rightarrow \cos^2\alpha=\frac{49}{50}$

$\cos\alpha=\sqrt{\frac{49}{50}}=\frac{7}{5\sqrt{2}}=\frac{7\sqrt{2}}{10}$ $\sin\alpha=\frac{1}{7}\times\frac{7}{\sqrt{50}}=\frac{1}{\sqrt{50}}=\frac{\sqrt{2}}{10}$

$\sin(\frac{\pi}{4}+\alpha) = \sin(\frac{\pi}{4})\cos\alpha + \cos(\frac{\pi}{4})\sin\alpha \Rightarrow \frac{7\sqrt{2}}{10} + \frac{\sqrt{2}}{10} = \frac{4\sqrt{2}}{5} \Rightarrow \sin(\frac{\pi}{4}+\alpha) \Rightarrow \frac{\sqrt{2}}{2}(\frac{4\sqrt{2}}{5}) = \frac{4\times(\sqrt{2}\times\sqrt{2})}{10} = \frac{8}{10} \Rightarrow -\frac{4}{5}$ (منفی)

-۵ $\sin^2 x + (\sin^2 x + \cos^2 x) = \frac{4}{3}$ $\quad \sin^2 x + \cos^2 x = 1 \xrightarrow{\text{پس}} \sin^2 x + 1 = \frac{4}{3}$

$\sin^2 x = \frac{4}{3} - 1 = \frac{4-3}{3} = \frac{1}{3}$ $\quad \cos^2 x = 1 - \sin^2 x = 1 - \frac{1}{3} = \frac{3-1}{3} = \frac{2}{3}$ (۲)

$\tan^2 x = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$ ✓

-۶ $S = \frac{1}{2} ab \sin\alpha \Rightarrow 4.5 = \frac{1}{2} \times 6 \times \sqrt{3} \times \sin\alpha$ (۲)

$4.5 = 3\sqrt{3}\sin\alpha \Rightarrow \sin\alpha = \frac{4.5}{3\sqrt{3}} = \frac{\sqrt{3}}{2}$

$\alpha\ min = 60$ $\quad 180 - 60 = 120$

$\alpha\ max = 120$

$\frac{120^\circ}{60^\circ} = 2$ ✓

-۷ $\frac{a}{b} = \frac{2}{3}$ $\quad a = 2x,\ b = 3x$ $\quad 180 - 150 = 30$ (۲)

$\sin 150 = \sin(180 - 30) = \sin 30 = \frac{1}{2}$

$54 = (2x)\times(3x)\times\sin(150) \Rightarrow 54 = 6x^2 \times \frac{1}{2} \Rightarrow 54 = 3x^2$

$x^2 = \frac{54}{3} = 18$ $\quad x = 3\sqrt{2}$ $\quad a = 2\times 3\sqrt{2} = 6\sqrt{2}$, $\ b = 3\times 3\sqrt{2} = 9\sqrt{2}$

$P = 2(a+b) \Rightarrow 2(15\sqrt{2}) = 30\sqrt{2}$ ✓

-۸ $S_{ABC} = \frac{1}{2}\times AB \times AC \times \sin(A)$ $\quad S_{ABC}$ (۲)

$S_{ADE} = \frac{1}{2}\times AD \times AE \times \sin(A)$; ADE

$|S_{ABC} - S_{ADE}| = |17.5\sin(A) - 14\sin(A)| = 3.5\sin(A) = 1.75$

$= \frac{1.75}{3.5} = \frac{1}{2} = 0.5$ $\quad A = 30$ $\quad \tan(30) = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ ✓

-۹ $2BN = 3NC \Rightarrow BN = \frac{3}{2}NC$ $\quad BC = BN + NC = \frac{3}{2}NC + NC = \frac{5}{2}NC$ (۲)

$\frac{BN}{BC} = \frac{3}{5}$ $\quad \frac{S_{BMN}}{S_{ABC}} = \frac{BM\times BN}{AB\times BC} = x \times \frac{3}{5}$ $\quad \frac{S_{BMN}}{S_{ABC}} = \frac{1}{3}$

$\frac{BM}{AB} = x$ $\quad \frac{1}{3}\times\frac{5}{3} = \frac{5}{9}$ $\quad \frac{BM}{AB} = \frac{5}{9}$ $\quad AM = AB - BM = AB(1 - \frac{5}{9}) = \frac{4}{9}AB$

$\frac{BM = \frac{5}{9}}{AM = \frac{4}{9}} = \frac{5}{4}$ ✓

-۱۰ $\frac{1}{|\cos\alpha|} - \tan\alpha = \frac{1+\sin\alpha}{|\cos\alpha|} \Rightarrow -\tan\alpha = \frac{1+\sin\alpha-1}{|\cos\alpha|} \Rightarrow -\tan\alpha = \frac{\sin\alpha}{|\cos\alpha|}$ (۲)

$-\frac{\sin\alpha}{\cos\alpha} = \frac{\sin\alpha}{|\cos\alpha|}$ $\quad \sin \neq 0$ $\quad \frac{1}{\cos\alpha} = \frac{1}{|\cos\alpha|}$ $\quad \frac{1}{|\cos\alpha|} + \frac{1}{\cos\alpha} = 0$ ✓

ربع سوم

پاسخ ۲ و ۳ : $\frac{|\sin\alpha|}{\cos\alpha} = -\frac{\sin\alpha}{\cos\alpha}$ $\quad \cos\alpha \neq 0$ $\quad |\sin\alpha| = -\sin\alpha$

$\cos\alpha =$ منفی $\quad \sin\alpha =$ منفی