نام و نام خانوادگی ... پاسخنامه تشریحی تکلیف شماره ۱۸ کلاس ...

۱ (۲)

الف) $\tan\frac{11\pi}{4}+\sin\frac{15\pi}{4}\cos\frac{13\pi}{4}=\tan\left(2\pi+\frac{3\pi}{4}\right)+\sin\left(2\pi+\frac{7\pi}{4}\right)\cos\left(2\pi+\frac{5\pi}{4}\right)$

$=\tan\frac{3\pi}{4}+\sin\frac{7\pi}{4}\cos\frac{5\pi}{4}=-1+\left(-\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right)=-1+\frac{1}{2}=-\frac{1}{2}$ ✓

ب) $\tan\frac{17\pi}{6}\sin\frac{11\pi}{3}+\cos\frac{10\pi}{3}=\tan\left(2\pi+\frac{5\pi}{6}\right)\sin\left(2\pi+\frac{5\pi}{3}\right)+\cos\left(2\pi+\frac{4\pi}{3}\right)$

$=\tan\frac{5\pi}{6}\sin\frac{5\pi}{3}+\cos\frac{4\pi}{3}=-\frac{\sqrt{3}}{3}\left|-\frac{\sqrt{3}}{2}\right|+\left(-\frac{1}{2}\right)=\frac{1}{2}-\frac{1}{2}=0$ ✓

۲ (۲)

$\frac{3\cos\alpha-\sin\alpha}{\cos\alpha+\sin\alpha}=\frac{3\times\frac{4}{17}-\frac{1}{17}}{\frac{4}{17}+\frac{1}{17}}=\frac{\frac{11}{17}}{\frac{5}{17}}=\frac{11}{5}$ ✓

$\cot\alpha=4$

$\cot\alpha=\frac{\cos\alpha}{\sin\alpha}$

$\Rightarrow \sin\alpha=\frac{1}{17}$

$\cos\alpha=\frac{4}{17}$

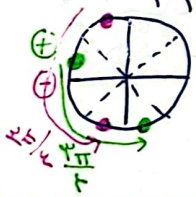
۳ (۲)

$\cos^2\alpha+\sin^2\alpha=1 \xrightarrow{\sin\alpha=2\cos\alpha} \cos^2\alpha+4\cos^2\alpha=5\cos^2\alpha=1$

$\Rightarrow \cos^2\alpha=\frac{1}{5}\xrightarrow{\text{از طرفین ریشه}}\cos\alpha=-\frac{1}{\sqrt{5}}=-\frac{\sqrt{5}}{5}$ ✓

۴ (۰/۵)

الف) $\cos\left(\frac{11\pi}{4}+\alpha\right)=\cos\left(2\pi+\frac{3\pi}{4}+\alpha\right)=\cos\left(\frac{3\pi}{4}+\alpha\right)=\pm\sin\alpha$



$\sin\alpha=\frac{\sqrt{2}}{10}<\frac{\sqrt{2}}{2}$

$\Rightarrow \sin\alpha=+$

$\Rightarrow$ ● $\rightarrow \cos\alpha=+$

$\Rightarrow \sin\alpha=+\frac{\sqrt{2}}{10}$

$\sin\left(\frac{13\pi}{4}+\alpha\right)=\sin\left(\frac{5\pi}{4}+\alpha\right)=\pm\cos\alpha$

$\tan^2\alpha+1=\frac{1}{\cos^2\alpha}\Rightarrow\frac{1}{49}+1=\frac{50}{49}=\frac{1}{\cos^2\alpha}\Rightarrow\cos\alpha=\frac{7}{\sqrt{50}}$

$\tan\alpha=\frac{1}{7}<1$

$\Rightarrow \alpha \rightarrow$ ● $\rightarrow \sin\alpha \rightarrow -$

$\sin\left(\frac{5\pi}{4}+\alpha\right)=-\cos\alpha=-\frac{7}{\sqrt{50}}$ ⇐

۵ (۲)

$2\sin^2\alpha+\cos^2\alpha=\frac{4}{3}$

$-\sin^2\alpha+-\cos^2\alpha=-1$

$\sin^2\alpha=\frac{1}{3}\Rightarrow\cos^2\alpha=\frac{2}{3}$

$\tan^2\alpha=\frac{\sin^2\alpha}{\cos^2\alpha}\xrightarrow[\cos^2\alpha=\frac{2}{3}]{\sin^2\alpha=\frac{1}{3}}\tan^2\alpha=\frac{\frac{1}{3}}{\frac{2}{3}}$

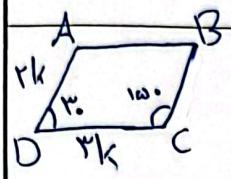
$=\frac{1}{2}$ ✓

$S_{ABC} = \frac{1}{2} \times 4 \times \sqrt{3} \times \sin\alpha = 1,5 \Rightarrow 2\sqrt{3}\sin\alpha = 1,5 \Rightarrow \sin\alpha = \frac{1,5}{2\sqrt{3}} = \frac{3}{4\sqrt{3}}$

$= \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$ ⟶ 60° $\sin 60 = \frac{\sqrt{3}}{2}$ ⟶ 120° $\sin 120 = \frac{\sqrt{3}}{2}$

$\Rightarrow \frac{120}{60} =$ دو برابر ✓

۶ (۲)



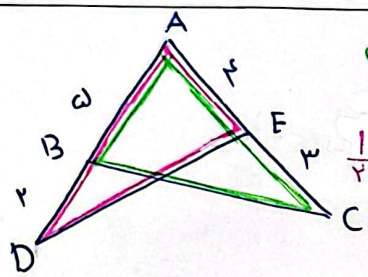
$S_{ABCD} = 24 \Rightarrow S_{ABCD} = \sin 150^\circ \times AD \times AC = 2K^2 \sqrt{3}$

$\Rightarrow K^2 = \frac{24}{\sqrt{3}} = 8 \Rightarrow K = \sqrt{8}$

$P_{ABCD} = AD + AB + DC + CB = 10K \xrightarrow{K=\sqrt{8}} P_{ABCD}$

$= 10\sqrt{8} = 20\sqrt{2}$ ✓

۷ (۲)

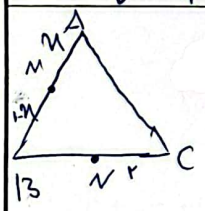


$S_{ABC}^{\triangle} - S_{ADE}^{\triangle} = 1,75 \Rightarrow \frac{1}{2} \times \sin A \times AB \times AC - \frac{1}{2} \times \sin A \times AD \times AE = \frac{3 \times 5}{2}\sin A - 4\sin A$

$= \frac{3 \times 5 - 2 \times 4}{2} \sin A = \frac{7}{2}\sin A = 1,75 \Rightarrow \sin A =$

$1,75 \times \frac{2}{7} = \frac{1}{2} \xrightarrow{\sin A = \frac{1}{2}} A = 30^\circ \Rightarrow \tan 30^\circ = \frac{\sqrt{3}}{3}$ ✓

۸ (۲)



$\frac{BN}{NC} = \frac{2}{3} \Rightarrow \frac{BN}{BC} = \frac{2}{5} \Rightarrow BN = \frac{2}{5} BC \rightarrow S_{ABC} = \frac{1}{2}$

مرتفع طول ضلع AB را یک در نظر گرفتیم

$S_{BMN} = \frac{1}{2}(BM)(MN) = \frac{1}{2} \times (1-m) \times \frac{2}{5} = \frac{1-m}{5}$

۹ (۵)

$\frac{1}{2} = 3 \times \frac{1-m}{5} \Rightarrow 1-m = \frac{5}{6} \Rightarrow m = \frac{1}{6} \Rightarrow \begin{matrix} BM = 1-m = \frac{5}{6} \\ AM = m = \frac{1}{6} \end{matrix} \Rightarrow \frac{BM}{AM} = \frac{5}{6} \times 6 = 5$

$\frac{|\sin\alpha|}{\cos\alpha} = -\frac{1}{\cot\alpha} \Rightarrow \frac{|\sin\alpha|}{\cos\alpha} = \frac{-\sin\alpha}{\cos\alpha} \Rightarrow |\sin\alpha|\cos\alpha = -\sin\alpha \cdot \cos\alpha$

$= -\tan\alpha \Rightarrow \sin\alpha =$ (−) ✓ ⟶ α در ناحیه سوم

$\frac{1}{\sqrt{\cos^2\alpha}} - \tan\alpha = \frac{1+\sin\alpha}{|\cos\alpha|} + \Rightarrow \frac{1}{|\cos\alpha|} - \frac{\sin\alpha}{\cos\alpha} =$ (+)

$\Rightarrow \cos\alpha =$ (−)

۱۰ (۱,۵)

$\frac{1}{|\cos\alpha|} - \frac{\sin\alpha}{\cos\alpha} = \frac{1+\sin\alpha}{|\cos\alpha|} \rightarrow -\sin\alpha = \sin\alpha \rightarrow \cos\alpha < 0$

۴) الف) $\cos^2\alpha = 1 - \sin^2\alpha \rightarrow \cos^2\alpha = \frac{۹۸}{۱۰۰} \xrightarrow{\text{ربع دوم}} \cos\alpha = -\frac{۷\sqrt{۲}}{۱۰}$

$\cos\left(\frac{۱۱\pi}{۴}+\alpha\right) = \cos\left(۳\pi+\left(-\frac{\pi}{۴}+\alpha\right)\right) = -\cos\left(\alpha-\frac{\pi}{۴}\right)$

$= -\left(\cos\alpha\cos\frac{\pi}{۴} + \sin\alpha\sin\frac{\pi}{۴}\right) = -\left(-\frac{۷\sqrt{۲}}{۱۰}\times\frac{\sqrt{۲}}{۲} + \frac{\sqrt{۲}}{۱۰}\times\frac{\sqrt{۲}}{۲}\right) = \frac{۳}{۵}$

ب) $1+\tan^2\alpha = \frac{1}{\cos^2\alpha} \xrightarrow{\text{ربع اول}} \cos\alpha = \frac{۷\sqrt{۲}}{۱۰} \xrightarrow{\sin^2\alpha+\cos^2\alpha=1} \sin\alpha = \frac{\sqrt{۲}}{۱۰}$

$\frac{1}{۴۹}$

$\sin\left(\frac{۱۳\pi}{۴}+\alpha\right) = \sin\left(۳\pi+\frac{\pi}{۴}+\alpha\right) = -\sin\left(\frac{\pi}{۴}+\alpha\right) \xrightarrow[\text{بالا}]{\text{مشابه مثال}} -\frac{۴}{۵}$

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۹)

$S_{\hat{ABC}} = \frac{1}{۲} AB \times ۵m \times \sin\hat{B}$

$S_{\hat{BMN}} = \frac{1}{۲} BM \times ۳m \times \sin\hat{B}$

$\rightarrow \frac{S_{\hat{ABC}}}{S_{\hat{BMN}}} = \frac{۵m \times AB}{۳m \times BM} = ۳$

$\frac{BM}{AB} = \frac{۵}{۹} \rightarrow \frac{BM}{AM+BM} = \frac{۵}{۹} \rightarrow \frac{BM}{AM} = \boxed{\frac{۵}{۴}}$

$۵m$ — $۴m$