نام و نام خانوادگی سلمان بهشت اللهی پاسخنامه تشریحی تکلیف شماره ..۱۷.. کلاس هم‌پیس A شنبه‌ها

۱

$\{1\}, \{2,3,4\}, \{5,6,7,8,9\}$

آخرین عدد : $1, 4, 9, \dots \Rightarrow K^2 \Rightarrow 9^2 = 81$ آخرین عدد دسته‌ی نهم

اولین عدد : $1, 2, 5, 10, \dots \Rightarrow (K-1)^2+1 \Rightarrow (9-1)^2+1 = 65$ اولین عدد دسته‌ی نهم

واسطه‌ی حسابی $= \dfrac{81+65}{2} = \boxed{73}$

۲

$\underset{1}{\{1,2,3\}}, \underset{2}{\{4,5,\dots,12\}}, \underset{3}{\{13,14,\dots,39\}}, \underset{4}{\{40,41,\dots,120\}}, \underbrace{\underset{5}{\{121,122,\dots,363\}}}_{\text{دسته‌ی پنجم}}$

اعداد پشت سرهم قرار گرفتند.

میانگین $= \dfrac{121+363}{2} = \boxed{242}$

۳

$[\ ] \Rightarrow$ نماد جزء صحیح

$a_0 + a_1 + \dots + a_9 = (a_0 + a_3 + a_6 + a_9) + (a_1 + a_4 + a_7) + (a_2 + a_5 + a_8)$

$= (2^0 + 2^1 + 2^2 + 2^3) + (4+2+0) + (1+a+1+a+2+a) \Rightarrow 25 + 3a = 19$

$\Rightarrow a = -2 \Rightarrow a_{29} = \left[\dfrac{29}{9+2}\right] - 2 = 2 - 2 = 0 \quad , \quad a_2 = \left[\dfrac{2}{0+2}\right] - 2 = -1$

$a_5 = \left[\dfrac{5}{1+2}\right] - 2 = -1 \quad , \quad a_8 = \left[\dfrac{8}{2+2}\right] - 2 = 0 \quad , \quad a_{11} = \left[\dfrac{11}{3+2}\right] - 2 = 0$

$a_{14} = 0, \ a_{17} = 0, \ a_{20} = 0, \ a_{23} = 0, \ a_{26} = 0 \Rightarrow -1 + -1 = -2$ جواب نهایی

۴

$a_5 = 14 \xrightarrow[\text{قرینه}]{} -14 \xrightarrow[\frac{1}{70}]{} -0.2 \Rightarrow a_n = -0.2n^2 + bn + c$

$-0.2(25) + 5b + c = 14 \Rightarrow -5 + 5b + c = 14 \Rightarrow 5b + c = 19$

$-0.2(49) + 7b + c = 17.2 \Rightarrow -9.8 + 7b + c = 17.2 \Rightarrow 7b + c = 27$

$2b = 8 \Rightarrow b = 4$

$5(4) + c = 19 \Rightarrow c = -1$

$a_1 = -0.2(1) + 4 - 1 = 2.8$

$a_{15} = -0.2(225) + 15(4) = 14$

$\dfrac{a_{15}}{a_1} = \dfrac{14}{2.8} = 5$

۵

$a_n = a_1 + (n-1)d$ $a_4 = a_1 + 3d$ $a_8 = a_1 + 7d$	الگوی خطی $\Downarrow$ $L_n = pn + q$	$L_2 = a_4$ $L_7 = a_8 \quad \Rightarrow \dfrac{L_{15}}{d} = ?$ $L_{10} = 0$

$L_2 = a_4 = a_1 + 3d \Rightarrow 2p + q = a_1 + 3d$ (1)

$L_7 = a_8 = a_1 + 7d \Rightarrow 7p + q = a_1 + 7d$ (2)

$5p = 4d \Rightarrow p = \frac{4}{5}d$

$\xrightarrow{L_{10}=0} 10p + q = 0$ (3)

$10(\frac{4}{5}d) + q = 0 \Rightarrow 8d + q = 0 \Rightarrow q = -8d$

$\Rightarrow 2(\frac{4}{5}d) + q = a_1 + 3d \Rightarrow a_1 = -\frac{47}{5}d$

$L_{15} = 15(\frac{4}{5}r) - 8r = 4r \Rightarrow \frac{4r}{r} = 4$ جواب نهایی

$\Rightarrow \dfrac{L_{15}}{r} = \dfrac{4r}{r} = 4$ جواب نهایی

$a_1 = a \;,\; a_2 = a+d \;,\; a_3 = a+2d \;,\; a_4 = a+3d$

$6(a_2)^2 = 5(a_3 \times a) + 3(a_2 \times a) \Rightarrow 6(a+d)^2 = 5a(a+2d) + 3a(a+d)$

$6a^2 + 12ad + 6d^2 = 8a^2 + 13ad \Rightarrow 6a^2 + 12ad + 6d^2 - 8a^2 - 13ad = 0$

$\Rightarrow -2a^2 - ad + 6d^2 = 0 \Rightarrow 2a^2 + ad - 6d^2 = 0 \xrightarrow{\div d^2} 2\left(\frac{a}{d}\right)^2 + \left(\frac{a}{d}\right) - 6 = 0$ ۶

$x = \frac{a}{d} \Rightarrow 2x^2 + x - 6 = 0 \Rightarrow x = \frac{-1 \pm \sqrt{1+48}}{4} \Rightarrow x \begin{cases} \frac{3}{2} \\ -2 \end{cases} \Rightarrow \frac{a}{d} = \frac{3}{2}$ یا $\frac{a}{d} = -2$

$a_4 = a + 3d = d\left(\frac{a}{d} + 3\right) \rightarrow a_4 = d\left(\frac{3}{2} + 3\right) = d\left(\frac{9}{2}\right)$

$a_4 = d(-2+3) = d(1) = d$ $\quad \frac{a_4}{d} = \frac{9}{2} \quad \frac{a_4}{d} = 1 \quad \boxed{4/5 \,,\, 1}$

$q = \frac{\frac{a}{2}}{\frac{c}{4}} = \frac{a}{2} \times \frac{4}{c} = \frac{2a}{c} \quad , \quad b = \left(\frac{c}{4}\right)q^2$

$b = \frac{a+c}{2} \Rightarrow \frac{a+c}{2} = \frac{c}{4}q^2 \Rightarrow q = \frac{2a}{c} \Rightarrow q^2 = \frac{4a^2}{c^2}$

$\frac{a+c}{2} = \frac{c}{4} \times \frac{4a^2}{c^2} \Rightarrow \frac{a+c}{2} = \frac{a^2}{c} \xRightarrow{\times 2c} c(a+c) = 2a^2 \xrightarrow{\div c^2} x = \frac{a}{c}$ ۷

$2x^2 - x - 1 = 0 \Rightarrow x = \frac{1 \pm \sqrt{1+8}}{4} \begin{cases} x = 1 \times \text{ غ‌ق‌ق} \\ x = -\frac{1}{2} \end{cases} \Longrightarrow \frac{a}{c} = -\frac{1}{2}$

$q = \frac{2a}{c} = 2\left(-\frac{1}{2}\right) = -1 \rightarrow -1 \times 2 = -2$

$a, b, c \rightarrow b = ar, c = ar^2$

$c, 2a, 3b \rightarrow 2a - c = 3b - 2a \Rightarrow 2a - ar^2 = 3(ar) - 2a \Rightarrow 4a - ar^2 - 3ar = 0 \xrightarrow{\div a \neq 0}$

$4 - r^2 - 3r = 0 \Rightarrow r^2 + 3r - 4 = 0 \Rightarrow r = \frac{-3 \pm \sqrt{9+16}}{2} = \frac{-3 \pm 5}{2} \begin{cases} r = 1 \times \text{ غ‌ق‌ق} \\ r = -4 \end{cases}$

$a_n = ar^{n-1}$ ۸

$\frac{a_9}{a_6} = r^3 \Longrightarrow r^3 = (-4)^3 = \boxed{-64}$

$\frac{a_6}{a_2 \times a_2 \times a_2} + \frac{a_2}{a \times a} = 2 \Rightarrow \frac{ar^5}{(ar)^3} + \frac{ar}{a^2} = 2 \longrightarrow \frac{r^2}{a^2} + \frac{r}{a} = 2$

$x = \frac{r}{a} \longrightarrow x^2 + x = 2 \Rightarrow x^2 + x - 2 = 0 \longrightarrow x = 1$ یا $x = -2$

$\frac{r}{a} = 1$ یا $\frac{r}{a} = -2 \longrightarrow \frac{a}{r} = 1$ یا $\frac{a}{r} = -\frac{1}{2}$ ۹

$\frac{a^2}{a_2} = \frac{a^2}{ar} = \frac{a}{r} \begin{cases} -\frac{1}{2} \\ 1 \end{cases}$

$a_5 = 27 \rightsquigarrow \quad \frac{1}{3} \xrightarrow{\times 3} 1 \xrightarrow{\times 3} 3 \xrightarrow{\times 3} 9 \xrightarrow{\times 3} 27 \longrightarrow$ جمله‌ی سوم جمله‌ی چهارم

$\frac{1}{2} - \frac{1}{3} = \frac{3-2}{6} = \boxed{\frac{1}{6}}$ ۱۰