

۱. k :

$$\{(k-1)^2 + 1, (k-1)^2 + 2, \dots, k^2\}$$

$$\frac{1}{q} = (1-1)^2 + 1 = 1^2 + 1 = 2$$

$$\frac{1}{q} = 1^2 = 1$$

$$\text{میانگین دانه} = \frac{2a+1}{2} = \frac{1+5}{2} = 3$$

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$$s_k = \{s_1, s_2, \dots, s_k\} \Rightarrow e_k = 3s_k \Rightarrow s_{k+1} \Rightarrow e_{k+1} = 3s_{k+1} \Rightarrow s_1 = 1$$

$$s_1 = 1 \Rightarrow s_2 = 2 \Rightarrow s_3 = 3 \Rightarrow s_4 = 4 \Rightarrow s_5 = 121$$

$$e_5 = 3s_5 = 3 \times 121 = 363 \Rightarrow \text{میانگین دانه} = \frac{1+363}{2} = 182$$

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$$a_k = 2^k / a_{k+1} = -2k+2 / a_{k+2} = \left[\frac{n}{k+2} \right]$$

$$k \geq 1 \rightarrow 2, 3, 4, \dots \text{ (درج)} \Rightarrow a_{k+2} = \left[\frac{2k+2}{k+2} \right] = 2$$

$$k \geq 0 \rightarrow 1, 2, 3, \dots \text{ (درج)} \Rightarrow a_{k+2} = \left[\frac{2k+2}{k+2} \right] = 2$$

$$k \geq 0 \rightarrow \text{درج صفر} \Rightarrow a_{k+2} = \left[\frac{2k+2}{k+2} \right] = 2$$

$$a_1 + a_2 + \dots + a_{29} \Rightarrow \text{درج صفر} \Rightarrow a_{k+2} = \left[\frac{2k+2}{k+2} \right] = 2$$

$$a_1 + a_2 + \dots + a_{29} = 1 + 1 + \dots + 1 = 29$$

$$a_1 = -2x_0 + 2 = 2 / a_2 = k = 0 \Rightarrow \left[\frac{2}{2} \right] + a_2 = 1 + a_2 = 2 \Rightarrow a_2 = 1$$

$$1 + a_2 = 2 \Rightarrow a_2 = 1 \Rightarrow a_3 = 2 \Rightarrow a_4 = 3 \Rightarrow a_5 = 4 \Rightarrow a_6 = 5 \Rightarrow a_7 = 6 \Rightarrow a_8 = 7 \Rightarrow a_9 = 8 \Rightarrow a_{10} = 9 \Rightarrow a_{11} = 10 \Rightarrow a_{12} = 11 \Rightarrow a_{13} = 12 \Rightarrow a_{14} = 13 \Rightarrow a_{15} = 14 \Rightarrow a_{16} = 15 \Rightarrow a_{17} = 16 \Rightarrow a_{18} = 17 \Rightarrow a_{19} = 18 \Rightarrow a_{20} = 19 \Rightarrow a_{21} = 20 \Rightarrow a_{22} = 21 \Rightarrow a_{23} = 22 \Rightarrow a_{24} = 23 \Rightarrow a_{25} = 24 \Rightarrow a_{26} = 25 \Rightarrow a_{27} = 26 \Rightarrow a_{28} = 27 \Rightarrow a_{29} = 28 \Rightarrow a_{30} = 29$$

$$a_1 = k = 2 \Rightarrow \left[\frac{2}{2} \right] + a_2 = 1 + a_2 = 2 \Rightarrow a_2 = 1$$

$$a_1 = -2x_3 + 2 = -2 \Rightarrow a_2 = 1 \Rightarrow a_3 = 2 \Rightarrow a_4 = 3 \Rightarrow a_5 = 4 \Rightarrow a_6 = 5 \Rightarrow a_7 = 6 \Rightarrow a_8 = 7 \Rightarrow a_9 = 8 \Rightarrow a_{10} = 9 \Rightarrow a_{11} = 10 \Rightarrow a_{12} = 11 \Rightarrow a_{13} = 12 \Rightarrow a_{14} = 13 \Rightarrow a_{15} = 14 \Rightarrow a_{16} = 15 \Rightarrow a_{17} = 16 \Rightarrow a_{18} = 17 \Rightarrow a_{19} = 18 \Rightarrow a_{20} = 19 \Rightarrow a_{21} = 20 \Rightarrow a_{22} = 21 \Rightarrow a_{23} = 22 \Rightarrow a_{24} = 23 \Rightarrow a_{25} = 24 \Rightarrow a_{26} = 25 \Rightarrow a_{27} = 26 \Rightarrow a_{28} = 27 \Rightarrow a_{29} = 28 \Rightarrow a_{30} = 29$$

$$a_1 = -2x_3 + 2 = -2 \Rightarrow a_2 = 1 \Rightarrow a_3 = 2 \Rightarrow a_4 = 3 \Rightarrow a_5 = 4 \Rightarrow a_6 = 5 \Rightarrow a_7 = 6 \Rightarrow a_8 = 7 \Rightarrow a_9 = 8 \Rightarrow a_{10} = 9 \Rightarrow a_{11} = 10 \Rightarrow a_{12} = 11 \Rightarrow a_{13} = 12 \Rightarrow a_{14} = 13 \Rightarrow a_{15} = 14 \Rightarrow a_{16} = 15 \Rightarrow a_{17} = 16 \Rightarrow a_{18} = 17 \Rightarrow a_{19} = 18 \Rightarrow a_{20} = 19 \Rightarrow a_{21} = 20 \Rightarrow a_{22} = 21 \Rightarrow a_{23} = 22 \Rightarrow a_{24} = 23 \Rightarrow a_{25} = 24 \Rightarrow a_{26} = 25 \Rightarrow a_{27} = 26 \Rightarrow a_{28} = 27 \Rightarrow a_{29} = 28 \Rightarrow a_{30} = 29$$

$$a_1 = -2x_3 + 2 = -2 \Rightarrow a_2 = 1 \Rightarrow a_3 = 2 \Rightarrow a_4 = 3 \Rightarrow a_5 = 4 \Rightarrow a_6 = 5 \Rightarrow a_7 = 6 \Rightarrow a_8 = 7 \Rightarrow a_9 = 8 \Rightarrow a_{10} = 9 \Rightarrow a_{11} = 10 \Rightarrow a_{12} = 11 \Rightarrow a_{13} = 12 \Rightarrow a_{14} = 13 \Rightarrow a_{15} = 14 \Rightarrow a_{16} = 15 \Rightarrow a_{17} = 16 \Rightarrow a_{18} = 17 \Rightarrow a_{19} = 18 \Rightarrow a_{20} = 19 \Rightarrow a_{21} = 20 \Rightarrow a_{22} = 21 \Rightarrow a_{23} = 22 \Rightarrow a_{24} = 23 \Rightarrow a_{25} = 24 \Rightarrow a_{26} = 25 \Rightarrow a_{27} = 26 \Rightarrow a_{28} = 27 \Rightarrow a_{29} = 28 \Rightarrow a_{30} = 29$$

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$$a_n = 14 \Rightarrow A = \frac{1}{n} \times \left(\frac{-1}{n} \right) = -\frac{1}{n^2} \Rightarrow a_n = An^2 + Bn + C \Rightarrow A = -\frac{1}{n^2} \Rightarrow \frac{1}{n^2} \times 10 + 5B + C = 14$$

$$a_v = 14 \Rightarrow \frac{1}{v} \times \left(\frac{-1}{v} \right) = -\frac{1}{v^2} \Rightarrow a_v = An^2 + Bn + C \Rightarrow A = -\frac{1}{v^2} \Rightarrow \frac{1}{v^2} \times 10 + 5B + C = 14$$

$$= 14/2 \Rightarrow -1/1 + 5B + C = 14/2 \Rightarrow 5B + C = 14/2 + 1 = 16/2 = 8 \Rightarrow 5B + C = 8 \Rightarrow B = 1 \Rightarrow C = 8 - 5 = 3 \Rightarrow a_n = -\frac{1}{n^2} + n + 3$$

$$a_1 = -\frac{1}{1^2} + 1 + 3 = 3 \Rightarrow a_2 = -\frac{1}{2^2} + 2 + 3 = 3 - \frac{1}{4} = \frac{11}{4} \Rightarrow a_3 = -\frac{1}{3^2} + 3 + 3 = 3 - \frac{1}{9} = \frac{26}{9} \Rightarrow a_4 = -\frac{1}{4^2} + 4 + 3 = 4 - \frac{1}{16} = \frac{63}{16} \Rightarrow a_5 = -\frac{1}{5^2} + 5 + 3 = 5 - \frac{1}{25} = \frac{124}{25} \Rightarrow a_6 = -\frac{1}{6^2} + 6 + 3 = 6 - \frac{1}{36} = \frac{215}{36} \Rightarrow a_7 = -\frac{1}{7^2} + 7 + 3 = 7 - \frac{1}{49} = \frac{342}{49} \Rightarrow a_8 = -\frac{1}{8^2} + 8 + 3 = 8 - \frac{1}{64} = \frac{511}{64} \Rightarrow a_9 = -\frac{1}{9^2} + 9 + 3 = 9 - \frac{1}{81} = \frac{728}{81} \Rightarrow a_{10} = -\frac{1}{10^2} + 10 + 3 = 10 - \frac{1}{100} = \frac{999}{100}$$

$$a_{10} = -\frac{1}{10^2} + 10 + 3 = 10 - \frac{1}{100} = \frac{999}{100} \Rightarrow a_{11} = -\frac{1}{11^2} + 11 + 3 = 11 - \frac{1}{121} = \frac{1330}{121} \Rightarrow a_{12} = -\frac{1}{12^2} + 12 + 3 = 12 - \frac{1}{144} = \frac{1727}{144} \Rightarrow a_{13} = -\frac{1}{13^2} + 13 + 3 = 13 - \frac{1}{169} = \frac{2186}{169} \Rightarrow a_{14} = -\frac{1}{14^2} + 14 + 3 = 14 - \frac{1}{196} = \frac{2755}{196} \Rightarrow a_{15} = -\frac{1}{15^2} + 15 + 3 = 15 - \frac{1}{225} = \frac{3374}{225} \Rightarrow a_{16} = -\frac{1}{16^2} + 16 + 3 = 16 - \frac{1}{256} = \frac{4095}{256} \Rightarrow a_{17} = -\frac{1}{17^2} + 17 + 3 = 17 - \frac{1}{289} = \frac{4886}{289} \Rightarrow a_{18} = -\frac{1}{18^2} + 18 + 3 = 18 - \frac{1}{324} = \frac{5725}{324} \Rightarrow a_{19} = -\frac{1}{19^2} + 19 + 3 = 19 - \frac{1}{361} = \frac{6608}{361} \Rightarrow a_{20} = -\frac{1}{20^2} + 20 + 3 = 20 - \frac{1}{400} = \frac{7599}{400}$$

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$$A_n = A_1 + (n-1)d \Rightarrow A_r = 14 \Rightarrow A_n = 14$$

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$$1. P + 9 = 0 \Rightarrow P = -9$$

$$A_1 + 3d = 2P + 9, A_1 + 5d = 3P + 9 \Rightarrow 2d = 5P \Rightarrow P = \frac{2}{5}d, 9 = -1 \cdot P = -\frac{2}{5}d \Rightarrow d = -\frac{45}{2} \Rightarrow P = -45$$

$$P = \frac{2}{5}d, 9 = -1 \cdot P = -\frac{2}{5}d \Rightarrow d = -\frac{45}{2} \Rightarrow P = -45$$

$$L_{10} = 10P + 4 = 10 \times \left(-\frac{45}{2} \right) + 4 = -225 + 4 = -221 \Rightarrow \frac{L_{10}}{d} = \frac{-221}{-\frac{45}{2}} = \frac{442}{45}$$

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$$a_n = a + (n-1)d / s_{a,r} = \omega a_{\mu} + r_{n,r} a = g(a_r) = \omega a_{\mu} x_a + r_{n,r} x_a$$

$$a^r = a + d / a^r = a + rd \Rightarrow g(a+d)^r = g(a+rd)/xq + r(a+d)/xa \Rightarrow g(a^r + r^2 ad + d^2) = g(a^r) + 1^2 ad + g(d^2) \Rightarrow g(a^r) + 1^2 ad + r^2 a^r = 1^2 a^r + 1^2 ad \Rightarrow g(a^r) + 1^2 ad + g(d^2) = 1^2 a^r + 1^2 ad$$

$$\Rightarrow 9a^2 - 12ad + 11ad - 13ad + 9d^2 = 0 \Rightarrow -4ad + 9d^2 = 0 \Rightarrow 4a^2 + ad = 9d^2 \quad \text{dividing by } d \Rightarrow a = \frac{9d}{4}$$

$$\frac{v}{d} = \frac{p}{d} = 21 \Rightarrow r_1^2 + r_2^2 = 9 \Rightarrow 1r_1^2 + r_2^2 - 9 = 0 \Rightarrow r_1 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{1+9}}{2} = \frac{-1 \pm \sqrt{10}}{2}$$

$$b = a + d, c = a + r d \quad \text{①} \quad \frac{a}{b} = \frac{c}{r} \Rightarrow \frac{a}{a+d} = \frac{a}{r} \Rightarrow ar = bc \Rightarrow ar = a(a+d) \Rightarrow r = a+d$$

$$v = \frac{\frac{a}{\frac{1}{v}}}{b} = \frac{a}{v(a + \frac{1}{v})} = \frac{a}{v(a + \frac{1}{v})} = \frac{a}{v(-\frac{1}{v} a)} = -1 \xrightarrow{\text{برابر طرف ضرب}} -1 \times v = -\frac{v}{1}$$

$$a, b = ar, c = ar^2 \mid \text{با } r \neq 0 \Rightarrow 2a - 3b = c - 2a \Rightarrow 4a - 3b = c \Rightarrow 4a - 3ar = ar^2 \Rightarrow 4a - 3ar = ar^2 \Rightarrow 4 - 3r = r^2 \Rightarrow r^2 + 3r - 4 = 0 \Rightarrow (r+4)(r-1) = 0 \Rightarrow r = -4, r = 1 \Rightarrow \text{با } r = 1 \Rightarrow r = -4$$

$$a_n = ar^{n-1} / a_5 = ar^4 / a_7 = ar / \frac{ar}{(ar)^r} + \frac{ar}{ar} = r \Rightarrow \frac{ar^5}{(ar)^r} + \frac{ar}{ar} = \frac{r^r}{ar} + \frac{r}{a} = r$$

$$x = \frac{r}{a} / x^r + x = r \Rightarrow x \in \{1, -r\} / \left(\frac{ar}{ar} \in \left\{1, -\frac{1}{r}\right\} \right)$$

$$q_n = ar^{n-1} / a_p = ar^r / a_r = ar^w / a_w = ar^t = rV / a_p = \sqrt{ar^r} \Rightarrow ar^r = \sqrt{ar^r} \Rightarrow ar^r = \sqrt{a} \sqrt{r^r} = \sqrt{a} \times r^{\frac{r}{2}} \xrightarrow[\text{a}]{\text{r}} ar^r = \sqrt{a} r^{\frac{r}{2}} \xrightarrow[r^{\frac{r}{2}}]{\div} \sqrt{a} (\sqrt{a} r^{\frac{r}{2}} - 1) = 0 \Rightarrow \dots$$

 $\sqrt{a} r^{\frac{r}{2}} = 1 \Rightarrow r^{\frac{r}{2}} = \frac{1}{\sqrt{a}} \Rightarrow r = \frac{1}{a} \Rightarrow ar^{\frac{1}{a}} = \frac{1}{a^{\frac{1}{a}}} = rV \Rightarrow a^{\frac{1}{a}} = \frac{1}{rV} \Rightarrow a = \frac{1}{r^V}$
 $a_r = \frac{1}{r} \longrightarrow \frac{1}{r} - \frac{1}{r} = \frac{1}{e}$