

۱)  $k$ :

$$\{(k-1)^2 + 1, (k-1)^2 + 2, \dots, k^2\}$$

$$\overline{a_1} = (1-1)^2 + 1 = 1^2 = 1$$

$$\overline{a_k} = k^2 = 11$$

$$\text{میانگین دانه} = \frac{9a+11}{2} = \frac{145}{2} = 72.5 \checkmark$$

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$$s_k = \{s_1, s_2, \dots, s_k\} \Rightarrow e_k = s_k \Rightarrow s_{k+1} \Rightarrow e_{k+1} = s_{k+1} \Rightarrow s_1 = 1$$

$$s_1 = 1 \Rightarrow s_2 = 2 \Rightarrow s_3 = 3 \Rightarrow s_4 = 4 \Rightarrow s_5 = 11$$

$$e_5 = s_5 = 11 \Rightarrow 11/11 = 1 \Rightarrow \text{میانگین دانه} = \frac{1+2+3+4+11}{5} = 4.2 \checkmark$$

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$$a_k = 2^k / a_{k+1} = -2k+2 / a_{k+2} = \left[ \frac{n}{k+2} \right]$$

$$k \geq 1 \rightarrow 2, 3, 4, \dots \text{ (درج)} \Rightarrow a_{k+2} = \left[ \frac{2k+2}{k+2} \right]$$

$$k \geq 0 \rightarrow 1, 2, 3, \dots \text{ (درج)} \Rightarrow a_{k+2} = \left[ \frac{2k+2}{k+2} \right]$$

$$k \geq 0 \rightarrow \text{فرصت} \Rightarrow a_{k+2} = \left[ \frac{2k+2}{k+2} \right]$$

$$a_2 + a_3 + \dots + a_{29} \Rightarrow \text{برای } k=27 \Rightarrow a_{29} = 1$$

$$a_2 + a_3 + \dots + a_{29} = 1 + 1 + \dots + 1 = 28$$

$$a_2 + a_3 + \dots + a_{29} = 28$$

$$a_1 = -2x+2 / a_2 = k = 0 \Rightarrow \left[ \frac{2}{2} \right] + a_2 = 1 + a_2 = 1 \Rightarrow a_2 = 0$$

$$a_2 = 0 \Rightarrow a_3 = 2 / a_4 = -2x+2 / a_5 = -2x+2 = 0 \Rightarrow x=1$$

$$a_1 = k=2 \Rightarrow \left[ \frac{2}{2} \right] + a_2 = 1 + a_2 = 1 \Rightarrow a_2 = 0$$

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$$a_5 = 14$$

$$a_6 = 17/2$$

$$a_7 = 14/2$$

$$a_8 = -1/5 n^2 + 2n - 1$$

$$a_1 = -1/5 + 2 - 1 = 3/5$$

$$a_5 = -1/5 + 10 - 1 = 8$$

$$A = \frac{1}{5} \times \left( \frac{-14}{-5} \right) = \frac{1}{5}$$

$$a_n = An^2 + Bn + C \Rightarrow A = -1/5 \Rightarrow -1/5 n^2 + Bn + C = 14$$

$$\Rightarrow -1/5 + 2B + C = 14 \Rightarrow 2B + C = 14 + 1/5 = 71/5$$

$$\Rightarrow 2B = 14 \Rightarrow B = 7 \Rightarrow C = 14 - 7 = 7$$

$$\Rightarrow \frac{a_5}{a_1} = \frac{14}{3/5} = 23.3 \checkmark$$

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$$A_n = A_1 + (n-1)d$$

$$A_n = Pn + Q$$

$$A_n = Pn + Q$$

$$A_n = Pn + Q$$

$$\Rightarrow A_r = P_r$$

$$\Rightarrow A_n = P_n$$

$$\Rightarrow A_n = P_n$$

$$\Rightarrow A_n = P_n$$

$$1. P + Q = 0 \Rightarrow Q = -1. P$$

$$A_1 + 2d = 2P + Q, A_1 + 3d = 3P + Q \Rightarrow 2d = 5P$$

$$\Rightarrow P = \frac{2}{5} d, Q = -1. P = -2/5 d$$

$$\Rightarrow \frac{A_5}{d} = \frac{14}{2} = 7 \checkmark$$

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$$a_n = a + (n-1)d / \text{في } a_r = a + (r-1)d = a + r d - d = a + r d - d = a + (r-1)d$$

$$a_r = a + d / a_r = a + r d \Rightarrow \text{في } (a+d)^r = a(a+r d) + r(a+d) \Rightarrow \text{في } (a+d)^r = a^2 + r a d + r a + r d$$

$$9a^2 + 12ad + 9d^2 \Rightarrow 9a^2 + 12ad + 9d^2 = 9a^2 + 12ad \Rightarrow 9a^2 + 12ad + 9d^2 = 9a^2 + 12ad$$

$$\Rightarrow 9a^2 - 9a^2 + 12ad - 12ad + 9d^2 = 0 \Rightarrow -12ad + 9d^2 = 0 \Rightarrow 12ad = 9d^2 / a = a + d$$

$$\frac{a^2}{d} = \frac{r}{a} = x \Rightarrow 12x^2 + x = 9 \Rightarrow 12x^2 + x - 9 = 0 \Rightarrow x = \frac{-1 \pm \sqrt{1 + 432}}{24} = \frac{-1 \pm 21}{24}$$

$$x = \frac{20}{24} = \frac{5}{6}, x = \frac{-22}{24} = -\frac{11}{12} \Rightarrow \frac{a}{d} = \frac{5}{6} \text{ or } -\frac{11}{12} \Rightarrow \frac{a}{d} \in \{1, -\frac{1}{r}\} \checkmark$$

$$b = a + d, c = a + r d \text{ (1) } \frac{a}{b} = \frac{c}{r} \Rightarrow \frac{a}{a+d} = \frac{a+r d}{r} \Rightarrow a^2 = (a+d)(a+r d) \Rightarrow a^2 = a^2 + r a d + a d + r d^2 \Rightarrow r a d + a d + r d^2 = 0$$

$$r a d + a d + r d^2 = 0 \Rightarrow a d (r + 1) + r d^2 = 0 \Rightarrow a d (r + 1) = -r d^2 \Rightarrow a = -\frac{r d}{r + 1}$$

$$r = \frac{a}{b} = \frac{a}{a+d} = \frac{a}{a - \frac{r d}{r + 1}} = \frac{a}{a - \frac{1}{r}} = -1 \Rightarrow r = -1 \checkmark$$

$$a, b = ar, c = ar^2 / \text{في } a, b, c \Rightarrow r a - r b = c - r a \Rightarrow r a - r b = ar^2 - r a \Rightarrow r a - r a r = ar^2 - r a \Rightarrow r a - r a r = ar^2 \Rightarrow r - r r = r^2 \Rightarrow r + r r = 0$$

$$\Rightarrow (r + r)(-1 + r) = 0 \Rightarrow r = -1, r = 1 \Rightarrow r = 1 \Rightarrow r = -r$$

$$\left. \begin{array}{l} a_9 = ar^8 \\ a_5 = ar^4 \end{array} \right\} \Rightarrow \frac{a_9}{a_5} = \frac{ar^8}{ar^4} = r^4 \Rightarrow r^4 = -r^4 = -9 \checkmark$$

$$a_n = ar^{n-1} / a_5 = ar^4 / a_r = ar / \frac{a_5}{(ar)^r} + \frac{ar}{a^r} = r \Rightarrow \frac{ar^5}{(ar)^r} + \frac{ar}{a^r} = \frac{r^5}{ar} + \frac{r}{a} = r$$

$$x = \frac{r}{a} / x^r + x = r \Rightarrow x \in \{1, -\frac{1}{r}\} / \frac{a^r}{ar} \in \{1, -\frac{1}{r}\} \checkmark$$

$$a_n = ar^{n-1} / a_r = ar^r / a_5 = ar^5 / a_d = ar^5 = r v / a_r = \sqrt{ar} \Rightarrow ar^5 = \sqrt{ar} \Rightarrow ar^5 = \sqrt{a} \sqrt{r} \Rightarrow \sqrt{a} \sqrt{r} = \sqrt{a} \sqrt{r} \frac{r}{a} \Rightarrow ar^5 = \sqrt{a} \sqrt{r} \frac{r}{a} \Rightarrow \sqrt{a} (\sqrt{a} \sqrt{r} - 1) = 0 \Rightarrow \sqrt{a} \sqrt{r} = 1$$

$$\sqrt{a} \sqrt{r} = 1 \Rightarrow r^{\frac{1}{2}} = \frac{1}{\sqrt{a}} \Rightarrow r = \frac{1}{a} \Rightarrow ar \frac{1}{ar} = \frac{1}{a^r} = r v \Rightarrow a^r = \frac{1}{r v} \Rightarrow a = \frac{1}{r} \checkmark$$

$$a = \frac{1}{r} \checkmark \rightarrow \frac{1}{r} - \frac{1}{r} = \frac{1}{r} \checkmark$$

ضابطه اول  $\rightarrow a_0, a_3, a_4, a_9$

ضابطه دوم  $\rightarrow a_1, a_4, a_7$

ضابطه سوم  $\rightarrow a_2, a_5, a_8$

$$S_{11} = 1 + 4 + (1+a) + 2 + (1+a) + 4 + 0 + (2+a) + 8 = 19 \rightarrow a = -2$$

جمع عملیات خواسته شده از ضابطه سوم بدست میاد :

$$a_n = \left\lfloor \frac{n}{K+2} \right\rfloor - 2 \xrightarrow{n=3K+2} \left\lfloor \frac{3K+2}{K+2} \right\rfloor - 2 = \overset{\text{میاد بیرون}}{\left\lfloor 2 + \frac{K-2}{K+2} \right\rfloor} - 2 = \left\lfloor \frac{K-2}{K+2} \right\rfloor$$

این عبارت به ازای  $K=0$  برابر ۱- و به ازای بقیه اعداد صحفه پس جواب خواسته شده ۲-