به نام خدا

محدثه مومنی

(1)

$a_1+a_1q+a_1q^2=21$, $a_1^3q^3=64 \to a_1q=4$ $\} \to \frac{4}{q}+4+4q=21 \to 4\left(\frac{1}{q}+q\right)=17 \to \frac{1+q^2}{q}=\frac{17}{4}$

$\Rightarrow 4q^2-17q+4=0 \to q=\frac{1}{4}$ ، $q=4$ ✗ حسابی نباشد

در نتیجه $\Rightarrow a_1=16$, $a_2=4$, $a_3=1$

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(2)

$\frac{x^2+4}{2x}=\frac{2x}{x^2-2}=\frac{1}{q} \to 4x^2=x^4-2x^2+4x^2-8 \to x^2(x^2-2)=8$

$x \to +2$ ، -2 ✗

$a_1=8$, $a_2=4$, $a_3=2$ $\quad q=\times\frac{1}{2}$, $\times\frac{1}{2}$

$S=8\left(\frac{1-(\frac{1}{2})^7}{1-\frac{1}{2}}\right)=\frac{127}{8}$

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(3)

$S_5=a_1+a_1q+a_1q^2+a_1q^3+a_1q^4 \to a_1(1+q+q^2+q^3+q^4) \Rightarrow 243\times\frac{121}{81}=363$

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(4)

$q=\frac{64-1}{5+1}=10.5\,d \qquad a_4=$ عدد چهارم $=a_1+3d=1+31.5=32.5=A$

$A+B=32.5+8=40.5$

$q^{5+1}=\frac{64}{1} \Rightarrow q=2 \qquad a_4=a_1q^3 \to B=8$

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(5)

$-24, -\frac{95}{4}, \dots \to d=0.25 \to a_{101}=a_1+100d=-24+(100\times0.25)=1$

$128, a_2, \dots \to a_8=a_1q^7=1 \to 128\times q^7=1 \to q^7=\frac{1}{128} \to q=\frac{1}{2}$

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(6)

$a_3=b_1$, $a_7=b_2$, $a_9=b_3$ $\} \to b_2^2=b_1b_3 \to (a_1+6d)^2=(a_1+2d)(a_1+8d) \to$

$\cancel{a^2}+\cancel{36d^2}+12ad=\cancel{a^2}+8ad+\cancel{2ad}+\cancel{16d^2} \to 2d(10d+a)=0$

$20d^2+2ad$

$d=-\frac{a}{10}$ ، $d=0$ ✗ ثابت نباشد

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(7)

$a_2=b_1$, $a_4=b_2$, $a_8=b_3$ $\} \to b_2^2=b_1b_3 \to (a_1+3d)^2=(a_1+d)(a_1+7d) \to -2ad+2d^2=0 \to 2d(-a+d)=0$

$\to d=0$ ✗ ثابت نباشد ، $d=a$

$a_2=2a$, $a_4=4a$, $a_8=8a$ $\times 2$ $\quad K_{10}=a_1q^9 \to \frac{1}{4}\times2^9=128$

(8)

$3aq \,,\, 2aq^2 \,,\, aq^3 \to 2aq^2 - 3aq = aq^3 - 2aq^2 \to aq^2(q-2) = aq(2q-3)$

$\to q^2 - 4q + 3 = 0$

$q = 1$ ✗ نباید ثابت

$q = 3$

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(9)

$d_{\text{حسابی}} = -0/25 \to a_4 = 2 - 0/75 = 1/25$

$a_8 = 2 - 1/75 = 0/25$

$a_{13} = 2 - 3 = -1$

$\to (a_8 + k)^2 = (a_{13} + k)(a_4 + k) \to k = -5/25$

هندسی $= -9/25 \,,\, -5 \,,\, -4 \to q = \dfrac{\cancel{-9/25}}{-\cancel{5}} = \dfrac{-5}{-4} = \cancel{\;} = 1/25$

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(10)

$a + aq^3 + aq^6 = 73 \to a(1 + q^3 + q^6) = 73$

$d = aq^3 - a_1$

$\cancel{\ldots \to q^6 - 2q + 4 = 0}$

$a_1 + a_1 + d + a_1 + 9d = 73$

$3a_1 + 10d = 73$

$10aq^3 - 7a_1 = 73$

$a(1 + q^3 + q^6) = 73$

$\to a(9q^3 - 7) = a(1 + q^6) \to 9aq^3 - 7a_1 = a_1 + a_1q^3 + aq^6$

$\to 9q^3 - q^6 - 8 = 0 \to$

$q = 1$ ✗ نباید ثابت

$q = 2 \to a_1(1 + 8 + 64) = 73 \to \boxed{a_1 = 1}$

$d = aq^3 - a_1 \to (1 \times 8) - 1 = 7$